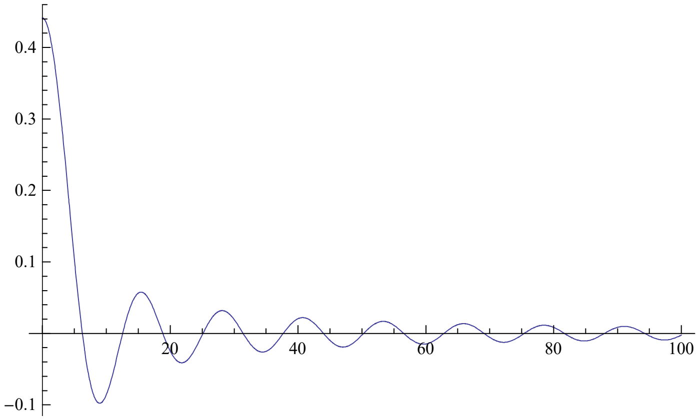
In[1]:=
```
Plot[BesselJ[0, 0.5 γ] BesselY[0, γ] - BesselJ[0, γ] BesselY[0, 0.5 γ] == 0,
 {γ, 0, 100}, PlotRange → All]
```

Out[1]=



In[2]:=
```
Table[FindRoot[
  BesselJ[0, 0.5 γ] BesselY[0, γ] - BesselJ[0, γ] BesselY[0, 0.5 γ] == 0, {γ, 6.2 n}], {n, 1, 20}]
```

FindRoot::lstol :
The line search decreased the step size to within tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the merit function. You may need more than MachinePrecision digits of working precision to meet these tolerances. ≫

Out[2]=
```
{{γ → 6.24606}, {γ → 12.5469}, {γ → 18.8364}, {γ → 25.1228}, {γ → 31.408},
 {γ → 37.6925}, {γ → 43.9766}, {γ → 50.2605}, {γ → 56.5443}, {γ → 62.8279},
 {γ → 69.1114}, {γ → 75.3949}, {γ → 81.6783}, {γ → 87.9618}, {γ → 94.2451},
 {γ → 100.528}, {γ → 106.812}, {γ → 113.095}, {γ → 119.378}, {γ → 125.662}}
```

In[3]:=

γ_{1} = 6.246061839191384;
γ_{2} = 12.546871427984357;
γ_{3} = 18.836415084503113;
γ_{4} = 25.122846371050727;
γ_{5} = 31.407995785488076;
γ_{6} = 37.692496076576745;
γ_{7} = 43.97662295096325;
γ_{8} = 50.26051551281339;
γ_{9} = 56.544251468059684;
γ_{10} = 62.82787760847569;
γ_{11} = 69.11142378494364;
γ_{12} = 75.39490993385962;
γ_{13} = 81.6783498758547;
γ_{14} = 87.96175349294495;
γ_{15} = 94.24512803776776;
γ_{16} = 100.52847895328937;
γ_{17} = 106.81181040394178;
γ_{18} = 113.09512563011177;
γ_{19} = 119.37842719090892;
γ_{20} = 125.66171713423309;

In[23]:=
```
Table[Xn[ξ_] = BesselJ[0, γn ξ] BesselY[0, γn] - BesselJ[0, γn] BesselY[0, γn ξ], {n, 1, 20}]
```

Out[23]=
```
{-0.23785 BesselJ[0, 6.24606 ξ] - 0.212219 BesselY[0, 6.24606 ξ],
 -0.163832 BesselJ[0, 12.5469 ξ] - 0.154462 BesselY[0, 12.5469 ξ],
 -0.132516 BesselJ[0, 18.8364 ξ] - 0.127378 BesselY[0, 18.8364 ξ],
 -0.114211 BesselJ[0, 25.1228 ξ] - 0.110865 BesselY[0, 25.1228 ξ],
 -0.101856 BesselJ[0, 31.408 ξ] - 0.0994589 BesselY[0, 31.408 ξ],
 -0.0928002 BesselJ[0, 37.6925 ξ] - 0.0909751 BesselY[0, 37.6925 ξ],
 -0.085796 BesselJ[0, 43.9766 ξ] - 0.0843471 BesselY[0, 43.9766 ξ],
 -0.0801704 BesselJ[0, 50.2605 ξ] - 0.0789841 BesselY[0, 50.2605 ξ],
 -0.0755234 BesselJ[0, 56.5443 ξ] - 0.074529 BesselY[0, 56.5443 ξ],
 -0.0716007 BesselJ[0, 62.8279 ξ] - 0.0707516 BesselY[0, 62.8279 ξ],
 -0.0682318 BesselJ[0, 69.1114 ξ] - 0.0674958 BesselY[0, 69.1114 ξ],
 -0.0652977 BesselJ[0, 75.3949 ξ] - 0.0646516 BesselY[0, 75.3949 ξ],
 -0.0627121 BesselJ[0, 81.6783 ξ] - 0.0621391 BesselY[0, 81.6783 ξ],
 -0.0604112 BesselJ[0, 87.9618 ξ] - 0.0598985 BesselY[0, 87.9618 ξ],
 -0.0583463 BesselJ[0, 94.2451 ξ] - 0.0578839 BesselY[0, 94.2451 ξ],
 -0.0564796 BesselJ[0, 100.528 ξ] - 0.0560599 BesselY[0, 100.528 ξ],
 -0.0547813 BesselJ[0, 106.812 ξ] - 0.054398 BesselY[0, 106.812 ξ],
 -0.0532275 BesselJ[0, 113.095 ξ] - 0.0528757 BesselY[0, 113.095 ξ],
 -0.0517988 BesselJ[0, 119.378 ξ] - 0.0514745 BesselY[0, 119.378 ξ],
 -0.0504794 BesselJ[0, 125.662 ξ] - 0.050179 BesselY[0, 125.662 ξ]}
```

In[24]:=
```
Table[An = NIntegrate[(1 - ξ^2 - (1 - 0.5^2) * Log[ξ]/Log[0.5]) Xn[ξ] ξ, {ξ, 0.5, 1}] /
   (NIntegrate[Xn[ξ]^2, {ξ, 0.5, 1}]), {n, 1, 20}]
```

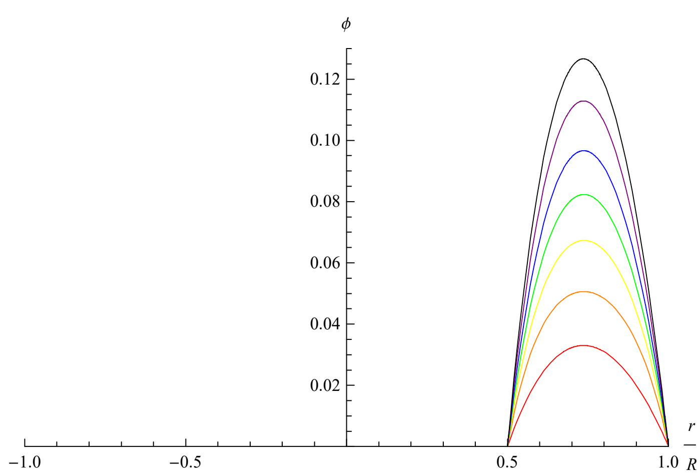
Out[24]=
```
{0.820535, 0.0339392, 0.0876006, 0.00842657, 0.0314227, 0.0037402, 0.0160158,
 0.00210288, 0.00968454, 0.00134555, 0.00648166, 0.000934303, 0.00464015,
 0.000686378, 0.003485, 0.000525484, 0.0027131, 0.000415184, 0.0021719, 0.000336292}
```

In[25]:=

$$\phi[\xi\_,\ \tau\_] = 1 - \xi^2 - (1 - 0.5^2) * \frac{\text{Log}[\xi]}{\text{Log}[0.5]} - \sum_{n=1}^{20} A_n \,\text{Exp}[-\gamma_n^2\, \tau]\, X_n[\xi];$$

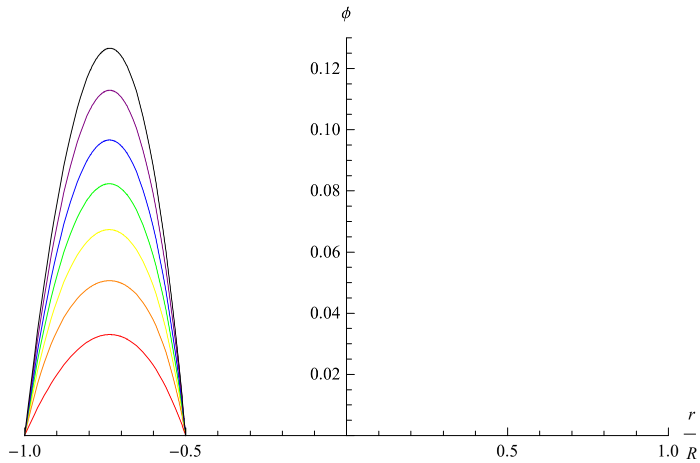
In[26]:=
```
Plot[{ϕ[Abs[ξ], 0], ϕ[Abs[ξ], 0.006], ϕ[Abs[ξ], 0.0125], ϕ[Abs[ξ], 0.02], ϕ[Abs[ξ], 0.03],
  ϕ[Abs[ξ], 0.05], ϕ[Abs[ξ], 5]}, {ξ, 0.5, 1}, PlotRange → {{-1, 1}, {0, 0.13}},
 AxesLabel → {r/R, ϕ}, PlotStyle → {Red, Orange, Yellow, Green, Blue, Purple, Black}]
```

Out[26]=



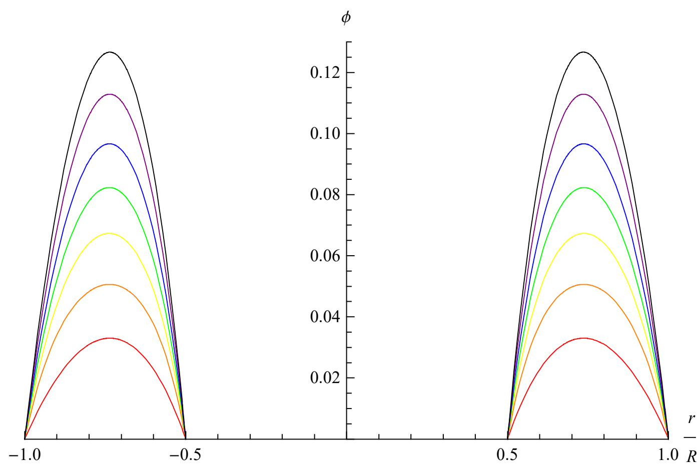
In[27]:=
```
Plot[{ϕ[Abs[ξ], 0], ϕ[Abs[ξ], 0.006], ϕ[Abs[ξ], 0.0125], ϕ[Abs[ξ], 0.02], ϕ[Abs[ξ], 0.03],
  ϕ[Abs[ξ], 0.05], ϕ[Abs[ξ], 5]}, {ξ, -1, -0.5}, PlotRange → {{-1, 1}, {0, 0.13}},
 AxesLabel → {r/R, ϕ}, PlotStyle → {Red, Orange, Yellow, Green, Blue, Purple, Black}]
```

Out[27]=



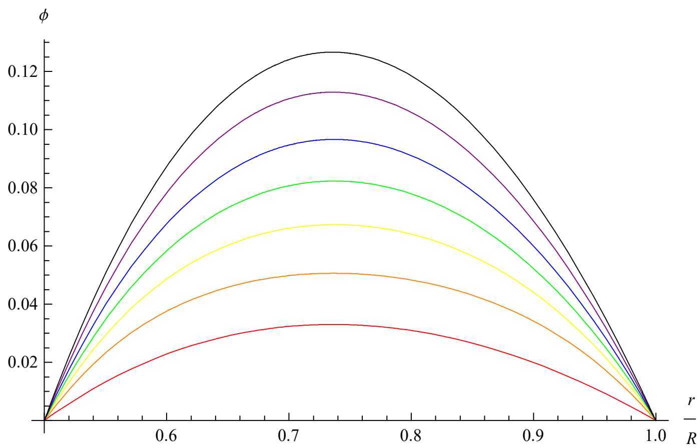
In[28]:=
```
Show[%, %%]
```

Out[28]=



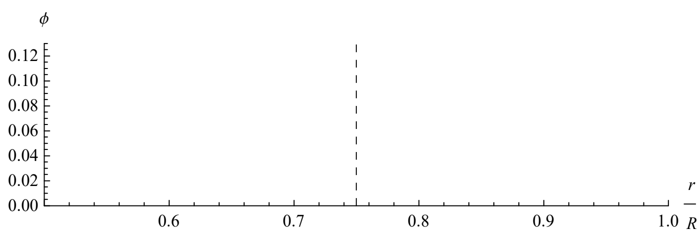
In[29]:=
```
Plot[{ϕ[Abs[ξ], 0], ϕ[Abs[ξ], 0.006], ϕ[Abs[ξ], 0.0125], ϕ[Abs[ξ], 0.02],
  ϕ[Abs[ξ], 0.03], ϕ[Abs[ξ], 0.05], ϕ[Abs[ξ], 5]}, {ξ, 0.5, 1}, PlotRange → All,
 AxesLabel → {r/R, ϕ}, PlotStyle → {Red, Orange, Yellow, Green, Blue, Purple, Black}]
```

Out[29]=



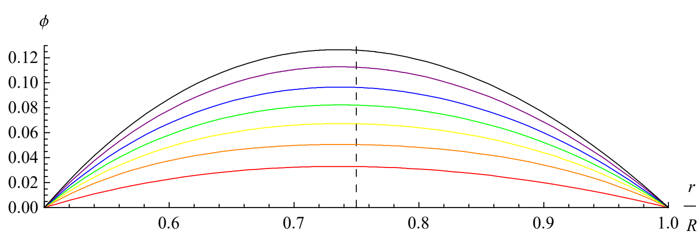
In[30]:=
```
ParametricPlot[{0.75, y}, {y, 0, 0.13}, PlotRange → {{0.5, 1}, {0, 0.13}},
 AxesLabel → {r/R, ϕ}, PlotStyle → {Black, Dashed}]
```

Out[30]=



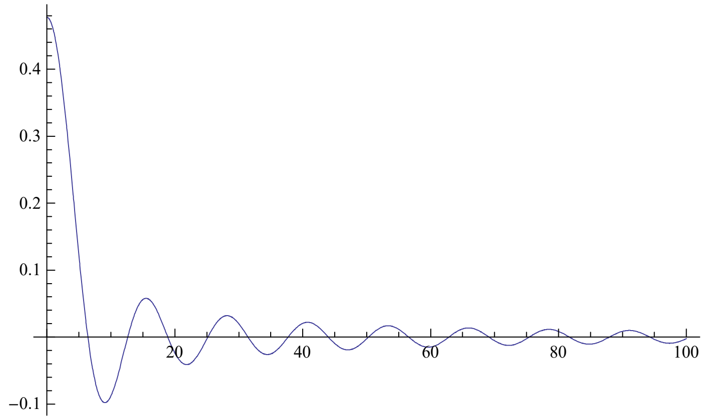
In[31]:=
```
Show[%, %%]
```

Out[31]=



In[32]:=
```
Plot[BesselJ[1, 0.5 β] BesselY[1, β] - BesselJ[1, β] BesselY[1, 0.5 β] == 0,
 {β, 0, 100}, PlotRange → All]
```

Out[32]=



In[33]:=
```
Table[FindRoot[
  BesselJ[1, 0.5 β] BesselY[1, β] - BesselJ[1, β] BesselY[1, 0.5 β] == 0, {β, 6.3 n}], {n, 1, 20}]
```

Out[33]=
```
{{β → 6.39316}, {β → 12.6247}, {β → 18.8889}, {β → 25.1624}, {β → 31.4397},
 {β → 37.719}, {β → 43.9993}, {β → 50.2804}, {β → 56.5619}, {β → 62.8438},
 {β → 69.1259}, {β → 75.4082}, {β → 81.6906}, {β → 87.9731}, {β → 94.2557},
 {β → 100.538}, {β → 106.821}, {β → 113.104}, {β → 119.387}, {β → 125.67}}
```

In[34]:=
β_1 = 6.393156761621268;
β_2 = 12.624699020746522;
β_3 = 18.888929850964466;
β_4 = 25.162405620208215;
β_5 = 31.439708538859477;
β_6 = 37.71895324027679;
β_7 = 43.99931604243465;
β_8 = 50.28038081375911;
β_9 = 56.5619149166635;
β_{10} = 62.843778196315014;
β_{11} = 69.12588121104642;
β_{12} = 75.40816421170508;
β_{13} = 81.69058577081975;
β_{14} = 87.9731162630476;
β_{15} = 94.2557339434426;
β_{16} = 100.53842249150883;
β_{17} = 106.82116941955894;
β_{18} = 113.10396501027418;
β_{19} = 119.38680158901695;
β_{20} = 125.66967301401135;

In[54]:=
```
Table[Fn[ξ_] = BesselJ[1, βn ξ] BesselY[1, βn] - BesselJ[1, βn] BesselY[1, βn ξ], {n, 1, 20}]
```

Out[54]=
```
{-0.258463 BesselJ[1, 6.39316 ξ] + 0.183494 BesselY[1, 6.39316 ξ],
 -0.172322 BesselJ[1, 12.6247 ξ] + 0.144394 BesselY[1, 12.6247 ξ],
 -0.137339 BesselJ[1, 18.8889 ξ] + 0.12197 BesselY[1, 18.8889 ξ],
 -0.117406 BesselJ[1, 25.1624 ξ] + 0.107383 BesselY[1, 25.1624 ξ],
 -0.104168 BesselJ[1, 31.4397 ξ] + 0.0969828 BesselY[1, 31.4397 ξ],
 -0.094571 BesselJ[1, 37.719 ξ] + 0.0890996 BesselY[1, 37.719 ξ],
 -0.0872082 BesselJ[1, 43.9993 ξ] + 0.0828637 BesselY[1, 43.9993 ξ],
 -0.0813304 BesselJ[1, 50.2804 ξ] + 0.077773 BesselY[1, 50.2804 ξ],
 -0.0764982 BesselJ[1, 56.5619 ξ] + 0.0735161 BesselY[1, 56.5619 ξ],
 -0.0724348 BesselJ[1, 62.8438 ξ] + 0.0698882 BesselY[1, 62.8438 ξ],
 -0.0689561 BesselJ[1, 69.1259 ξ] + 0.0667484 BesselY[1, 69.1259 ξ],
 -0.0659343 BesselJ[1, 75.4082 ξ] + 0.0639965 BesselY[1, 75.4082 ξ],
 -0.0632774 BesselJ[1, 81.6906 ξ] + 0.0615587 BesselY[1, 81.6906 ξ],
 -0.0609175 BesselJ[1, 87.9731 ξ] + 0.0593795 BesselY[1, 87.9731 ξ],
 -0.0588032 BesselJ[1, 94.2557 ξ] + 0.0574164 BesselY[1, 94.2557 ξ],
 -0.0568947 BesselJ[1, 100.538 ξ] + 0.0556357 BesselY[1, 100.538 ξ],
 -0.0551606 BesselJ[1, 106.821 ξ] + 0.054011 BesselY[1, 106.821 ξ],
 -0.0535758 BesselJ[1, 113.104 ξ] + 0.0525207 BesselY[1, 113.104 ξ],
 -0.0521202 BesselJ[1, 119.387 ξ] + 0.0511472 BesselY[1, 119.387 ξ],
 -0.0507771 BesselJ[1, 125.67 ξ] + 0.0498761 BesselY[1, 125.67 ξ]}
```

In[55]:=

$$\text{Table}\left[B_n = \frac{\text{NIntegrate}\left[\frac{\xi^2-0.5^2}{1-0.5^2}\,F_n[\xi],\ \{\xi,\ 0.5,\ 1\}\right]}{\text{NIntegrate}\left[F_n[\xi]^2\,\xi,\ \{\xi,\ 0.5,\ 1\}\right]},\ \{n,\ 1,\ 20\}\right]$$

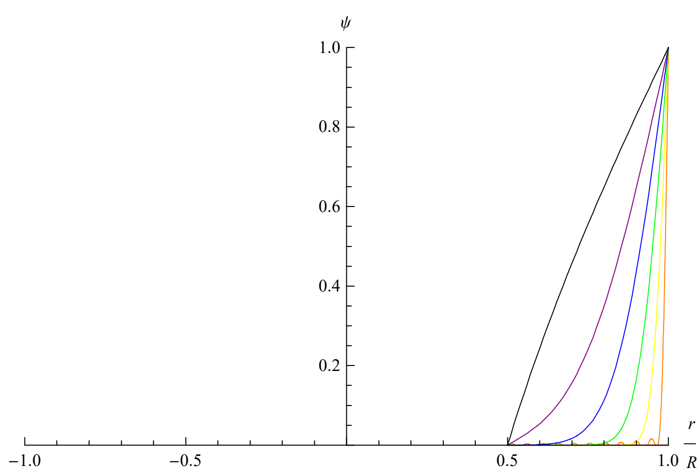
Out[55]=
```
{6.13382, 6.2407, 6.26376, 6.27215, 6.27609, 6.27824, 6.27955, 6.2804, 6.28098, 6.2814,
 6.28171, 6.28194, 6.28213, 6.28227, 6.28239, 6.28249, 6.28257, 6.28263, 6.28269, 6.28274}
```

In[56]:=

$$\psi[\xi\_,\ \tau\_] = \frac{\xi}{1-0.5^2}\left(1-\left(\frac{0.5}{\xi}\right)^2\right) - \sum_{n=1}^{20} B_n\,\text{Exp}\left[-\beta_n{}^2\,\tau\right]\,F_n[\xi];$$

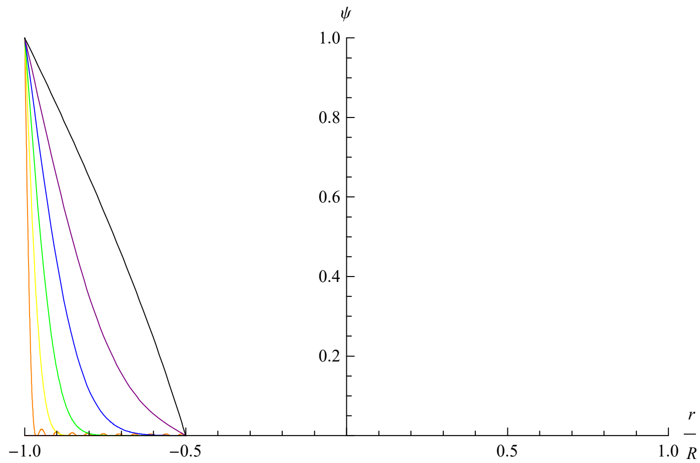
In[57]:=
```
Plot[{0, ψ[Abs[ξ], 0.0001], ψ[Abs[ξ], 0.0007], ψ[Abs[ξ], 0.0025], ψ[Abs[ξ], 0.0075],
  ψ[Abs[ξ], 0.02], ψ[Abs[ξ], 5]}, {ξ, 0.5, 1}, PlotRange → {{-1, 1}, {0, 1}},
 AxesLabel → {r/R, ψ}, PlotStyle → {Red, Orange, Yellow, Green, Blue, Purple, Black}]
```

Out[57]=



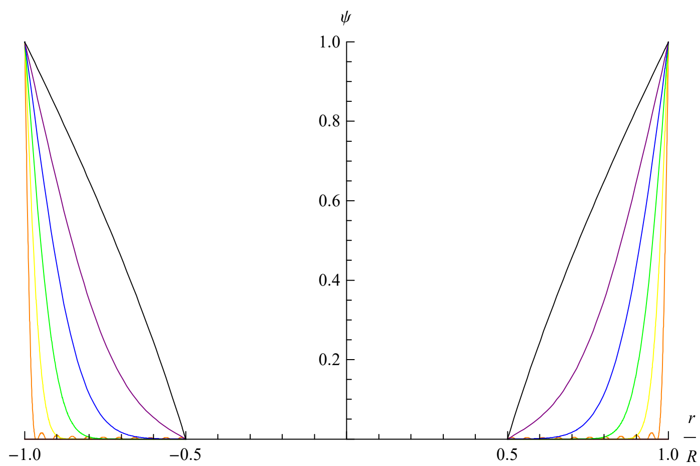
In[58]:=
```
Plot[{0, ψ[Abs[ξ], 0.0001], ψ[Abs[ξ], 0.0007], ψ[Abs[ξ], 0.0025], ψ[Abs[ξ], 0.0075],
  ψ[Abs[ξ], 0.02], ψ[Abs[ξ], 5]}, {ξ, -1, -0.5}, PlotRange → {{-1, 1}, {0, 1}},
 AxesLabel → {r/R, ψ}, PlotStyle → {Red, Orange, Yellow, Green, Blue, Purple, Black}]
```

Out[58]=



In[59]:=
```
Show[%, %%]
```

Out[59]=



In[60]:=
```
Plot[{0, ψ[Abs[ξ], 0.0001], ψ[Abs[ξ], 0.0007], ψ[Abs[ξ], 0.0025],
  ψ[Abs[ξ], 0.0075], ψ[Abs[ξ], 0.02], ψ[Abs[ξ], 5]}, {ξ, 0.5, 1}, PlotRange → All,
 AxesLabel → {r/R, ψ}, PlotStyle → {Red, Orange, Yellow, Green, Blue, Purple, Black}]
```

Out[60]=