Exercise 7.2.1

From Kirchhoff’s law the current $I$ in an $RC$ (resistance-capacitance) circuit (Fig. 7.1) obeys the equation

$$R \frac{dI}{dt} + \frac{1}{C} I = 0.$$

(a) Find $I(t)$.

(b) For a capacitance of $10,000 \, \mu F$ charged to $100 \, V$ and discharging through a resistance of $1 \, M\Omega$, find the current $I$ for $t = 0$ and for $t = 100$ seconds.

*Note.* The initial voltage is $I_0 R$ or $Q/C$, where $Q = \int_0^\infty I(t) \, dt$.

![RC circuit diagram](FIGURE 7.1 RC circuit.)

Solution

**Part (a)**

$$RI' + \frac{1}{C} I = 0$$

Bring the second term to the right side.

$$RI' = -\frac{1}{C} I$$

Divide both sides by $RI$.

$$\frac{I'}{I} = -\frac{1}{RC}$$

The left side can be written as $d/dt(\ln I)$ by the chain rule.

$$\frac{d}{dt}(\ln I) = -\frac{1}{RC}$$

Integrate both sides with respect to $t$.

$$\ln I = -\frac{t}{RC} + C_1$$

Exponentiate both sides.

$$I(t) = e^{-t/RC + C_1}$$

$$= e^{C_1}e^{-t/RC}$$

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Use a new constant $A$ for $e^{C_1}$.

$$I(t) = Ae^{-t/RC}$$

**Part (b)**

Use the relationship between voltage and charge for a capacitor to determine $A$.

$$V = \frac{Q}{C}$$

$$= \frac{1}{C} \int_{0}^{\infty} I(t) \, dt$$

$$= \frac{1}{C} \int_{0}^{\infty} Ae^{-t/RC} \, dt$$

$$= \frac{A}{C}(-RC)(0 - 1)$$

$$= AR$$

Consequently,

$$A = \frac{V}{R} = \frac{100 \text{ V}}{10^6 \Omega} = 0.0001 \text{ amps}$$

and

$$I(t) = 0.0001 \exp \left(-\frac{t}{10000}\right).$$

Therefore,

$$I(0) = 0.0001 \text{ amps}$$

$$I(100) \approx 0.000099 \text{ amps}.$$ 

The graph below shows $I(t)$ vs. $t$, the charge on the capacitor as a function of time.