Exercise 7.3.1

Find the general solutions to the following ODEs. Write the solutions in forms that are entirely real (i.e., that contain no complex quantities).

$$y''' - 2y'' - y' + 2y = 0.$$

Solution

Because this is a linear homogeneous ODE and all the coefficients on the left side are constant, the solutions for it are of the form $y = e^{rt}$.

$$y = e^{rt} \quad \rightarrow \quad y' = re^{rt} \quad \rightarrow \quad y'' = r^2 e^{rt} \quad \rightarrow \quad y''' = r^3 e^{rt}$$

Substitute these formulas into the ODE.

$$r^3 e^{rt} - 2(r^2 e^{rt}) - re^{rt} + 2(e^{rt}) = 0$$

Divide both sides by e^{rt}.

$$r^3 - 2r^2 - r + 2 = 0$$

Solve for r.

$$(r+1)(r-1)(r-2) = 0$$

$$r = \{-1, 1, 2\}$$

Three solutions to the ODE are $y = e^{-t}$ and $y = e^t$ and $y = e^{2t}$. By the principle of superposition, the general solution is a linear combination of these three. Therefore,

$$y(t) = C_1 e^{-t} + C_2 e^t + C_3 e^{2t}.$$