

Exercise 9.4.7

The 1-D Schrödinger wave equation for a particle in a potential field $V = \frac{1}{2}kx^2$ is

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + \frac{1}{2}kx^2\psi = E\psi(x).$$

(a) Defining

$$a = \left(\frac{mk}{\hbar^2}\right)^{1/4}, \quad \lambda = \frac{2E}{\hbar} \left(\frac{m}{k}\right)^{1/2},$$

and setting $\xi = ax$, show that

$$\frac{d^2\psi(\xi)}{d\xi^2} + (\lambda - \xi^2)\psi(\xi) = 0.$$

(b) Substituting

$$\psi(\xi) = y(\xi)e^{-\xi^2/2},$$

show that $y(\xi)$ satisfies the Hermite differential equation.