Exercise 9.7.1

For a homogeneous spherical solid with constant thermal diffusivity, K, and no heat sources, the equation of heat conduction becomes

$$\frac{\partial T(r,t)}{\partial t} = K\nabla^2 T(r,t).$$

Assume a solution of the form

$$T = R(r)T(t)$$

and separate variables. Show that the radial equation may take on the standard form

$$r^2\frac{d^2R}{dr^2} + 2r\frac{dR}{dr} + \alpha^2 r^2 R = 0,$$

and that $\sin\alpha r/r$ and $\cos\alpha r/r$ are its solutions.

[TYPO: T represents the temperature. Use a different variable Θ for the separated function of t.]