Exercise 2

Draw a sketch to illustrate the inequality in Eq. A.1-9. Are there any special cases for which it becomes an equality?

Solution

Eq. A.1-9 says that

$$(\mathbf{u}\cdot\mathbf{v})\mathbf{w} \neq \mathbf{u}(\mathbf{v}\cdot\mathbf{w}).$$

Using the definition of the dot product, we can write each side as

$$\begin{aligned}(\mathbf{u}\cdot\mathbf{v})\mathbf{w} &= (|\mathbf{u}||\mathbf{v}|\cos\phi_{uv})|\mathbf{w}|\hat{\mathbf{w}} = |\mathbf{u}||\mathbf{v}||\mathbf{w}|\cos\phi_{uv}\hat{\mathbf{w}} \\ \mathbf{u}(\mathbf{v}\cdot\mathbf{w}) &= |\mathbf{u}|\hat{\mathbf{u}}(|\mathbf{v}||\mathbf{w}|\cos\phi_{vw}) = |\mathbf{u}||\mathbf{v}||\mathbf{w}|\cos\phi_{vw}\hat{\mathbf{u}},\end{aligned}$$

where ϕ_{uv} is the angle between $\mathbf{u}$ and $\mathbf{v}$ and ϕ_{vw} is the angle between $\mathbf{v}$ and $\mathbf{w}$. For the two sides to be equal, $\mathbf{u}$ and $\mathbf{w}$ have to be parallel or antiparallel.

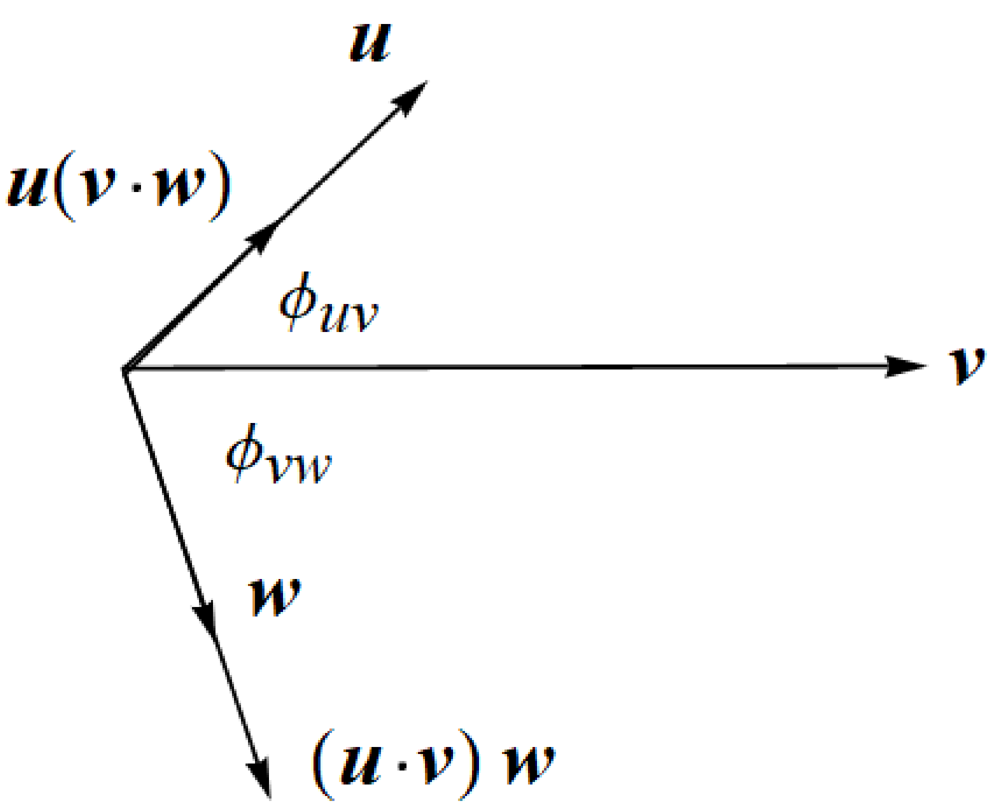

Figure 1: A sketch illustrating Eq. A.1-9.

$\mathbf{u}\cdot\mathbf{v}$ represents the product of $\mathbf{v}$'s magnitude and the component of $\mathbf{u}$ in the direction of $\mathbf{v}$. This product is the factor by which $\mathbf{w}$ is elongated. Similarly, $\mathbf{v}\cdot\mathbf{w}$ is the factor by which $\mathbf{u}$ is elongated (here in the sketch a shortening occurs because $\mathbf{v}$ and $\mathbf{w}$ are almost perpendicular). In general, these elongated vectors point in different directions from one another.