Exercise 1

What are the “orders” of the following quantities: \((v \cdot w), (v - u)w, (ab : cd), [v \cdot \rho w]u,\\[[a \times f] \times [b \times g]]\)?

Solution

The order of the quantities is determined by adding the orders of the tensors involved (scalar = 0, vector = 1, second-order tensor = 2, third-order tensor = 3, and so on) and subtracting numbers based on what operations are done (dot product = 2, cross product = 1, and double dot product = 4).

\[
\begin{align*}
(v \cdot w) & \rightarrow \text{Order} = 1 + 1 - 2 = 0 \\
(v - u)w & \rightarrow \text{Order} = 1 + 1 = 2 \\
(ab : cd) & \rightarrow \text{Order} = 1 + 1 + 1 + 1 - 4 = 0 \\
[v \cdot \rho w]u & \rightarrow \text{Order} = 1 + 1 + 1 - 2 = 1 \\
[a \times f] \times [b \times g] & \rightarrow \text{Order} = 1 - 1 + 1 - 1 + 1 - 1 + 1 = 1
\end{align*}
\]

For the second entry, \(v - u\) counts as one vector and \(w\) counts as the other.