Exercise 2

Draw a sketch to illustrate the inequality in Eq. A.1-9. Are there any special cases for which it becomes an equality?

Solution

Eq. A.1-9 says that

\[(u \cdot v)w \neq u(v \cdot w)\].

Using the definition of the dot product, we can write each side as

\[(u \cdot v)w = (|u||v|\cos \phi_{uv})|w|\hat{w} = |u||v||w|\cos \phi_{uv}\hat{w}\]
\[u(v \cdot w) = |u|\hat{u}(|v||w|\cos \phi_{vw}) = |u||v||w|\cos \phi_{vw}\hat{u},\]

where \(\phi_{uv}\) is the angle between \(u\) and \(v\) and \(\phi_{vw}\) is the angle between \(v\) and \(w\). For the two sides to be equal, \(u\) and \(w\) have to be parallel or antiparallel.

![Figure 1: A sketch illustrating Eq. A.1-9.](image)

\(u \cdot v\) represents the product of \(v\)'s magnitude and the component of \(u\) in the direction of \(v\). This product is the factor by which \(w\) is elongated. Similarly, \(v \cdot w\) is the factor by which \(u\) is elongated (here in the sketch a shortening occurs because \(v\) and \(w\) are almost perpendicular). In general, these elongated vectors point in different directions from one another.