

Exercise 1

Write out the following summations:

$$(a) \sum_{k=1}^3 k^2 \quad (b) \sum_{k=1}^3 a_k^2 \quad (c) \sum_{j=1}^3 \sum_{k=1}^3 a_{jk} b_{kj} \quad (d) \left(\sum_{j=1}^3 a_j \right)^2 = \sum_{j=1}^3 \sum_{k=1}^3 a_j a_k$$

Solution

$$(a) \sum_{k=1}^3 k^2 = 1^2 + 2^2 + 3^2 = 1 + 4 + 9 = 14$$

$$(b) \sum_{k=1}^3 a_k^2 = a_1^2 + a_2^2 + a_3^2 = |\mathbf{a}|^2$$

$$(c) \sum_{j=1}^3 \sum_{k=1}^3 a_{jk} b_{kj} = \sum_{j=1}^3 (a_{j1} b_{1j} + a_{j2} b_{2j} + a_{j3} b_{3j}) = \sum_{j=1}^3 a_{j1} b_{1j} + \sum_{j=1}^3 a_{j2} b_{2j} + \sum_{j=1}^3 a_{j3} b_{3j}$$

$$= a_{11} b_{11} + a_{21} b_{12} + a_{31} b_{13}$$

$$+ a_{12} b_{21} + a_{22} b_{22} + a_{32} b_{23}$$

$$+ a_{13} b_{31} + a_{23} b_{32} + a_{33} b_{33}$$

$$(d) \left(\sum_{j=1}^3 a_j \right)^2 = \sum_{j=1}^3 \sum_{k=1}^3 a_j a_k = \sum_{j=1}^3 (a_j a_1 + a_j a_2 + a_j a_3) = \sum_{j=1}^3 a_j a_1 + \sum_{j=1}^3 a_j a_2 + \sum_{j=1}^3 a_j a_3$$

$$= a_1 \sum_{j=1}^3 a_j + a_2 \sum_{j=1}^3 a_j + a_3 \sum_{j=1}^3 a_j$$

$$= a_1 (a_1 + a_2 + a_3)$$

$$+ a_2 (a_1 + a_2 + a_3)$$

$$+ a_3 (a_1 + a_2 + a_3)$$

$$= a_1^2 + a_2^2 + a_3^2 + 2a_1 a_2 + 2a_1 a_3 + 2a_2 a_3$$