Exercise 5

Consider a rigid structure composed of point particles joined by massless rods. The particles are numbered $1, 2, 3, \ldots, N$, and the particle masses are m_v $(v = 1, 2, \ldots, N)$. The locations of the particles with respect to the center of mass are $\mathbf{R}_v$. The entire structure rotates on an axis passing through the center of mass with an angular velocity $\mathbf{W}$. Show that the angular momentum with respect to the center of mass is

$$\mathbf{L} = \sum_v m_v[\mathbf{R}_v \times [\mathbf{W} \times \mathbf{R}_v]]$$

Then show that the latter expression may be rewritten as

$$\mathbf{L} = [\boldsymbol{\Phi} \cdot \mathbf{W}]$$

where

$$\boldsymbol{\Phi} = \sum_v m_v \{(\mathbf{R}_v \cdot \mathbf{R}_v)\boldsymbol{\delta} - \mathbf{R}_v\mathbf{R}_v\}$$

is the *moment-of-inertia tensor*.