Exercise 4

Verify that the entries for $\nabla^2\mathbf{v}$ in Table A.7-2 can be obtained by any one of the following methods:

(a) First verify that, in cylindrical coordinates the operator $(\nabla \cdot \nabla)$ is

$$(\nabla \cdot \nabla) = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \tag{A.7-36}$$

and then apply the operator to $\mathbf{v}$.

(b) Use the expression for $[\nabla \cdot \boldsymbol{\tau}]$ in Table A.7-2, but substitute the components for $\nabla\mathbf{v}$ in place of the components of $\boldsymbol{\tau}$, so as to obtain $[\nabla \cdot \nabla\mathbf{v}]$.

(c) Use Eq. A.4-22:

$$\nabla^2\mathbf{v} = \nabla(\nabla \cdot \mathbf{v}) - [\nabla \times [\nabla \times \mathbf{v}]] \tag{A.7-37}$$

and use the gradient, divergence, and curl operations in Table A.7-2 to evaluate the operations on the right side.