Problem 1C.1

Some consequences of the Maxwell-Boltzmann distribution. In the simplified kinetic theory in §1.4, several statements concerning the equilibrium behavior of a gas were made without proof. In this problem and the next, some of these statements are shown to be exact consequences of the Maxwell–Boltzmann velocity distribution.

The Maxwell–Boltzmann distribution of molecular velocities in an ideal gas at rest is

$$f(u_x, u_y, u_z) = n(m/2\pi\kappa T)^{3/2} \exp(-mu^2/2\kappa T) \tag{1C.1-1}$$

in which $\mathbf{u}$ is the molecular velocity, n is the number density, and $f(u_x, u_y, u_z)du_x du_y du_z$ is the number of molecules per unit volume that is expected to have velocities between u_x and $u_x + du_x$, u_y and $u_y + du_y$, u_z and $u_x + du_z$. It follows from this equation that the distribution of the molecular speed u is

$$f(u) = 4\pi n u^2 (m/2\pi\kappa T)^{3/2} \exp(-mu^2/2\kappa T) \tag{1C.1-2}$$

(a) Verify Eq. 1.4-1 by obtaining the expression for the mean speed $\bar{u}$ from

$$\bar{u} = \frac{\displaystyle\int_0^\infty u f(u) du}{\displaystyle\int_0^\infty f(u) du} \tag{1C.1-3}$$

(b) Obtain the mean values of the velocity components $\bar{u}_x$, $\bar{u}_y$, and $\bar{u}_z$. The first of these is obtained from

$$\bar{u}_x = \frac{\displaystyle\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u_x f(u_x, u_y, u_z) du_x du_y du_z}{\displaystyle\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(u_x, u_y, u_z) du_x du_y du_z} \tag{1C.1-4}$$

What can one conclude from these results?

(c) Obtain the mean kinetic energy per molecule from

$$\frac{1}{2} m\overline{u^2} = \frac{\displaystyle\int_0^\infty \frac{1}{2} m u^2 f(u) du}{\displaystyle\int_0^\infty f(u) du} \tag{1C.1-5}$$

The correct result is $\frac{1}{2} m\overline{u^2} = \frac{3}{2}\kappa T$.