Problem 1C.2

The wall collision frequency. It is desired to find the frequency Z with which the molecules in an ideal gas strike a unit area of a wall from one side only. The gas is at rest and at equilibrium with a temperature T and the number density of the molecules is n. All molecules have a mass m. All molecules in the region $x < 0$ with $u_x > 0$ will hit an area S in the yz-plane in a short time Δt if they are in the volume $Su_x\Delta t$. The number of wall collisions per unit area per unit time will be

$$\begin{aligned} Z &= \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{+\infty}(Su_x\Delta t)f(u_x,u_y,u_z)du_xdu_ydu_z}{S\Delta t} \\ &= n\left(\frac{m}{2\pi\kappa T}\right)^{3/2}\left(\int_0^{+\infty}u_x\exp(-mu_x^2/2\kappa T)\,du_x\right) \\ &\quad \left(\int_{-\infty}^{+\infty}\exp(-mu_y^2/2\kappa T)\,du_y\right)\left(\int_{-\infty}^{+\infty}\exp(-mu_z^2/2\kappa T)\,du_z\right) \\ &= n\sqrt{\frac{\kappa T}{2\pi m}} = \frac{1}{4}n\bar{u} \end{aligned} \tag{1C.2-1}$$

Verify the above development.