Problem 1C.3

Pressure of an ideal gas.[4] It is desired to get the pressure exerted by an ideal gas on a wall by accounting for the rate of momentum transfer from the molecules to the wall, assuming specular reflections.

(a) When a molecule traveling with a velocity **u** collides with a wall, its incoming velocity components are u_x, u_y, u_z, and after a specular reflection at the wall, its components are $-u_x$, u_y, u_z. Thus the net momentum transmitted to the wall by a molecule is $2mu_x$. The molecules that have an x-component of the velocity equal to u_x, and that will collide with the wall during a small time interval Δt, must be within the volume $Su_x\Delta t$. How many molecules with velocity components in the range from u_x, u_y, u_z to $u_x + \Delta u_x$, $u_y + \Delta u_y$, $u_z + \Delta u_z$ will hit an area S of the wall with a velocity u_x within a time interval Δt? It will be $f(u_x, u_y, u_z)du_x du_y du_z$ times $Su_x\Delta t$. Then the pressure exerted on the wall by the gas will be

$$p = \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{0}^{+\infty}(Su_x\Delta t)(2mu_x)f(u_x, u_y, u_z)du_x du_y du_z}{S\Delta t} \tag{1C.3-1}$$

Explain carefully how this expression is constructed. Verify that this relation is dimensionally correct.

(b) Insert Eq. 1C.1-1 for the Maxwell–Boltzmann equilibrium distribution into Eq. 1C.3-1 and perform the integration. Verify that this procedure leads to $p = n\kappa T$, the ideal gas law.

[4] R. J. Silbey and R. A. Alberty, *Physical Chemistry*, Wiley, New York, 3rd edition (2001), pp. 639-640.