Problem 2A.2

Determination of capillary radius by flow measurement. One method for determining the radius of a capillary tube is by measuring the rate of flow of a Newtonian liquid through the tube. Find the radius of a capillary from the following flow data:

Length of capillary tube	50.02 cm
Kinematic viscosity of liquid	4.03×10^{-5} m^2/s
Density of liquid	0.9552×10^{3} kg/m^3
Pressure drop in the horizontal tube	4.829×10^{5} Pa
Mass rate of flow through tube	2.997×10^{-3} kg/s

What difficulties may be encountered in this method? Suggest some other methods for determining the radii of capillary tubes.

Solution

The mass rate of flow is given by the Hagen-Poiseuille equation,

$$w = \frac{\pi(\mathscr{P}_0 - \mathscr{P}_L)R^4\rho}{8\mu L}.$$

Solve this equation for the radius R.

$$R^4 = \frac{8\mu L w}{\pi(\mathscr{P}_0 - \mathscr{P}_L)\rho}$$

Take the fourth root of both sides.

$$R = \sqrt[4]{\frac{8\mu L w}{\pi(\mathscr{P}_0 - \mathscr{P}_L)\rho}}$$

We want to write this in terms of kinematic viscosity ν, one of the given quantities. Use Eq. 1.1-3, $\nu = \mu/\rho$, on page 13.

$$= \sqrt[4]{\frac{8\nu L w}{\pi(\mathscr{P}_0 - \mathscr{P}_L)}}$$

Now plug in the numbers.

$$= \sqrt[4]{\frac{8(4.03 \times 10^{-5}\ \text{m}^2/\text{s})(0.5002\ \text{m})(2.997 \times 10^{-3}\ \text{kg/s})}{\pi(4.829 \times 10^{5}\ \text{Pa})}}$$

Therefore,

$$R \approx 7.51 \times 10^{-4}\ \text{m} \quad \text{or} \quad R \approx 0.751\ \text{mm}.$$

The problem with this method is that there are many assumptions that have to hold in order for the Hagen-Poiseuille equation to be valid. The assumptions are summarized on page 52: flow is laminar, density is constant, flow is steady, fluid is Newtonian, capillary length must be much larger than its radius, fluid behaves as a continuum, and no slip at capillary wall. To determine the radius of a capillary tube, measure it directly with a micrometer. If one is not available, find out what volume of liquid it takes to fill up the capillary tube and use $V = \pi R^2 L$ to find R.