Problem 2A.3

Volume flow rate through an annulus. A horizontal annulus, 27 ft in length, has an inner radius of 0.495 in. and an outer radius of 1.1 in. A 60% aqueous solution of sucrose ($C_{12}H_{22}O_{11}$) is to be pumped through the annulus at 20°C. At this temperature the solution density is 80.3 lb_m/ft^3 and the viscosity is 136.8 $\text{lb}_m/\text{ft}\cdot\text{hr}$. What is the volume flow rate when the impressed pressure difference is 5.39 psi?

Answer: 0.110 ft^3/s

Solution

We obtain the volume flow rate from the mass flow rate w.

$$w = \frac{dm}{dt} = \frac{d(\rho V)}{dt} = \rho\frac{dV}{dt}$$

Dividing both sides by the density ρ, we obtain the volume flow rate.

$$\frac{dV}{dt} = \frac{w}{\rho}.$$

The mass flow rate in an annulus is given by Eq. 2.4-17 on page 55.

$$w = \frac{\pi(\mathscr{P}_0 - \mathscr{P}_L)R^4\rho}{8\mu L}\left[(1-\kappa^4) - \frac{(1-\kappa^2)^2}{\ln(1/\kappa)}\right] \tag{2.4-17}$$

Thus,

$$\frac{dV}{dt} = \frac{\pi(\mathscr{P}_0 - \mathscr{P}_L)R^4}{8\mu L}\left[(1-\kappa^4) - \frac{(1-\kappa^2)^2}{\ln(1/\kappa)}\right].$$

Our aim now is to convert all the given quantities to SI units before we plug them into this formula. The conversion factors are in Table F.3-2 and Table F.3-4 on page 869 and page 870, respectively.

$$\mathscr{P}_0 - \mathscr{P}_L = 5.39\,\frac{\text{lb}_f}{\text{in}^2} \times \frac{6.8947\times10^3\ \text{Pa}}{1\,\frac{\text{lb}_f}{\text{in}^2}} \approx 3.72\times10^4\ \text{Pa}$$

$$\mu = 136.8\ \text{lb}_m/\text{ft}\cdot\text{hr} \times \frac{4.1338\times10^{-4}\ \text{Pa}\cdot\text{s}}{1\ \text{lb}_m/\text{ft}\cdot\text{hr}} \approx 0.05655\ \text{Pa}\cdot\text{s}$$

$$L = 27\ \text{ft} \times \frac{1\ \text{m}}{3.28\ \text{ft}} \approx 8.2\ \text{m}$$

$$R = 1.1\ \text{in} \times \frac{1\ \text{ft}}{12\ \text{in}} \times \frac{1\ \text{m}}{3.28\ \text{ft}} \approx 0.028\ \text{m}$$

The dimensionless quantity κ is the ratio of the inner radius to the outer radius.

$$\kappa = \frac{0.495\ \text{in}}{1.1\ \text{in}} = 0.45$$

Now substitute all these numbers into the formula. Therefore,

$$\frac{dV}{dt} \approx \frac{\pi(3.72\times10^4\ \text{Pa})(0.028\ \text{m})^4}{8(0.05655\ \text{Pa}\cdot\text{s})(8.2\ \text{m})}\left[(1-0.45^4) - \frac{(1-0.45^2)^2}{\ln(1/0.45)}\right]$$

$$\approx 0.00310\,\frac{\text{m}^3}{\text{s}} \times \left(\frac{3.28\ \text{ft}}{1\ \text{m}}\right)^3 \approx 0.110\,\frac{\text{ft}^3}{\text{s}}.$$