Problem 2A.4

Loss of catalyst particles in stack gas.

(a) Estimate the maximum diameter of microspherical catalyst particles that could be lost in the stack gas of a fluid cracking unit under the following conditions:

Gas velocity at axis of stack	= 1.0 ft/s (vertically upward)
Gas viscosity	= 0.026 cp
Gas density	= 0.045 lb_m/ft^3
Density of a catalyst particle	= 1.2 g/cm^3

Express the result in microns (1 micron = 10^{-6}m = 1μm).

(b) Is it permissible to use Stokes' law in (a)?

Answers: **(a)** 110 μm; Re = 0.93

Solution

Convert all the given quantities to SI units and assign variables to them. The conversion factors can be found on page 868 and 870.

$$\text{Gas velocity at axis of stack } v_\infty = 1.0\ \frac{\cancel{\text{ft}}}{\text{s}} \times \frac{1\ \text{m}}{3.28\ \cancel{\text{ft}}} \approx 0.3049\ \frac{\text{m}}{\text{s}}$$

$$\text{Gas viscosity } \mu = 0.026\ \cancel{\text{cp}} \times \frac{10^{-3}\ \text{Pa}\cdot\text{s}}{1\ \cancel{\text{cp}}} = 2.6\times10^{-5}\ \text{Pa}\cdot\text{s}$$

$$\text{Gas density } \rho_g = 0.045\ \frac{\cancel{\text{lb}_m}}{\cancel{\text{ft}^3}} \times \frac{1\ \text{kg}}{2.2046\ \cancel{\text{lb}_m}} \times \left(\frac{3.28\ \cancel{\text{ft}}}{1\ \text{m}}\right)^3 \approx 0.7203\ \frac{\text{kg}}{\text{m}^3}$$

$$\text{Density of a catalyst particle } \rho_c = 1.2\ \frac{\cancel{\text{g}}}{\cancel{\text{cm}^3}} \times \frac{1\ \text{kg}}{1000\ \cancel{\text{g}}} \times \left(\frac{100\ \cancel{\text{cm}}}{1\ \text{m}}\right)^3 = 1200\ \frac{\text{kg}}{\text{m}^3}$$

Part (a)

Gas particles in a stack flow upward. Those at the center of the stack travel at the highest velocity since they are furthest from the walls. Consequently, to find the maximum diameter of a catalyst particle, we consider one at the center of the stack. Catalysts that are lost do not rise up with the rest of the gas; rather, they fall at terminal velocity in the stack and have an acceleration of zero. The sum of the forces acting on one in the y-direction must be equal to zero. The gravitational force is pulling the catalysts down, and the buoyant and kinetic (drag) forces are pushing them up.

$$\begin{aligned}\sum F_y &= F - F_g = 0\\ &= \underbrace{\frac{4}{3}\pi R^3\rho_g g}_{\text{buoyant force}} + \underbrace{2\pi\mu R v_\infty}_{\text{form drag}} + \underbrace{4\pi\mu R v_\infty}_{\text{friction drag}} - \underbrace{\frac{4}{3}\pi R^3\rho_c g}_{\text{weight of catalyst}} = 0\\ &= \frac{4}{3}\pi R^3 g(\rho_g-\rho_c) + 6\pi\mu R v_\infty = 0\end{aligned}$$

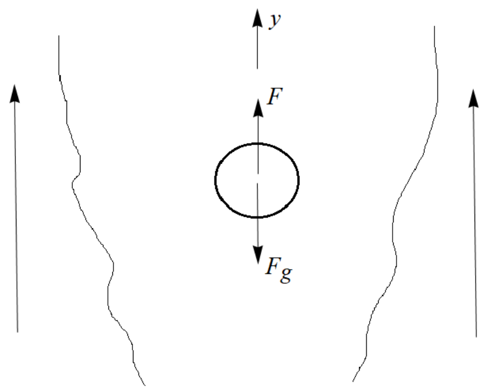

Figure 1: Free body diagram of a catalyst particle at the center of a stack. Define upward forces to be positive.

Solve this equation for the radius R.

$$\begin{aligned} R &= \sqrt{\frac{6\mu v_\infty}{g(\rho_c - \rho_g)} \cdot \frac{3}{4}} \\ &= \sqrt{\frac{9\mu v_\infty}{2g(\rho_c - \rho_g)}} \\ &\approx \sqrt{\frac{9(2.6 \times 10^{-5} \text{ Pa} \cdot \text{s})(0.3049 \text{ m/s})}{2(9.81 \text{ m/s}^2)(1200 - 0.7203) \text{ kg/m}^3}} \\ &\approx 5.5 \times 10^{-5} \text{ m} \end{aligned}$$

The diameter is twice the radius.

$$\begin{aligned} D = 2R &\approx 2(5.5 \times 10^{-5} \text{ m}) \\ &\approx 1.1 \times 10^{-4} \cancel{\text{m}} \times \frac{1 \text{ micron}}{10^{-6} \cancel{\text{m}}} = 110 \text{ microns} \end{aligned}$$

Part (b)

Calculate the Reynolds number.

$$\begin{aligned} \text{Re} = \frac{D v_\infty \rho_g}{\mu} &\approx \frac{(1.1 \times 10^{-4} \text{ m})(0.3049 \text{ m/s})(0.7203 \text{ kg/m}^3)}{(2.6 \times 10^{-5} \text{ Pa} \cdot \text{s})} \\ &\approx 0.93 \end{aligned}$$

Since the Reynolds number is less than 1, Stokes' law is valid.