Problem 2A.4

Loss of catalyst particles in stack gas.

(a) Estimate the maximum diameter of microspherical catalyst particles that could be lost in the stack gas of a fluid cracking unit under the following conditions:

- Gas velocity at axis of stack $v_\infty = 1.0 \text{ ft/s (vertically upward)}$
- Gas viscosity $\mu = 0.026 \text{ cp}$
- Gas density $\rho_g = 0.045 \text{ lb}_m/\text{ft}^3$
- Density of a catalyst particle $\rho_c = 1.2 \text{ g/cm}^3$

Express the result in microns (1 micron $= 10^{-6} \text{ m} = 1\mu\text{m}$).

(b) Is it permissible to use Stokes’ law in (a)?

**Answers:** (a) 110 $\mu\text{m}$; Re = 0.93

**Solution**

Convert all the given quantities to SI units and assign variables to them. The conversion factors can be found on page 868 and 870.

\[
\begin{align*}
\text{Gas velocity at axis of stack} & \quad v_\infty = 1.0 \text{ ft/s} \times \frac{1 \text{ m}}{3.28 \text{ ft}} \approx 0.3049 \text{ m/s} \\
\text{Gas viscosity} & \quad \mu = 0.026 \text{ cp} \times \frac{10^{-3} \text{ Pa} \cdot \text{s}}{1 \text{ cp}} = 2.6 \times 10^{-5} \text{ Pa} \cdot \text{s} \\
\text{Gas density} & \quad \rho_g = 0.045 \text{ lb}_m/\text{ft}^3 \times \frac{1 \text{ kg}}{2.2046 \text{ lb}_m} \times \left( \frac{3.28 \text{ ft}}{1 \text{ m}} \right)^3 \approx 0.7203 \text{ kg/m}^3 \\
\text{Density of a catalyst particle} & \quad \rho_c = 1.2 \text{ g/cm}^3 \times \frac{1 \text{ kg}}{1000 \text{ g}} \times \left( \frac{100 \text{ cm}}{1 \text{ m}} \right)^3 = 1200 \text{ kg/m}^3
\end{align*}
\]

**Part (a)**

Gas particles in a stack flow upward. Those at the center of the stack travel at the highest velocity since they are furthest from the walls. Consequently, to find the maximum diameter of a catalyst particle, we consider one at the center of the stack. Catalysts that are lost do not rise up with the rest of the gas; rather, they fall at terminal velocity in the stack and have an acceleration of zero. The sum of the forces acting on one in the $y$-direction must be equal to zero. The gravitational force is pulling the catalysts down, and the buoyant and kinetic (drag) forces are pushing them up.

\[
\sum F_y = F - F_y = 0
\]

\[
= \frac{4}{3} \pi R^3 \rho_g g + 2\pi \mu R v_\infty + 4\pi \mu R v_\infty - \frac{4}{3} \pi R^3 \rho_c g = 0
\]

\[
= \frac{4}{3} \pi R^3 g(\rho_g - \rho_c) + 6\pi \mu R v_\infty = 0
\]

www.stemjock.com
Figure 1: Free body diagram of a catalyst particle at the center of a stack. Define upward forces to be positive.

Solve this equation for the radius $R$.

$$R = \sqrt{\frac{6 \mu v_\infty}{g(\rho_c - \rho_g)}} \cdot \frac{3}{4}$$

$$= \sqrt{\frac{9 \mu v_\infty}{2g(\rho_c - \rho_g)}}$$

$$\approx \sqrt{\frac{9(2.6 \times 10^{-5} \text{ Pa} \cdot \text{s})(0.3049 \text{ m/s})}{2(9.81 \text{ m/s}^2)(1200 - 0.7203) \text{ kg/m}^3}}$$

$$\approx 5.5 \times 10^{-5} \text{ m}$$

The diameter is twice the radius.

$$D = 2R \approx 2(5.5 \times 10^{-5} \text{ m})$$

$$\approx 1.1 \times 10^{-4} \text{ m} \times \frac{1 \text{ micron}}{10^{-6} \text{ m}} = 110 \text{ microns}$$

**Part (b)**

Calculate the Reynolds number.

$$Re = \frac{D v_\infty \rho_g}{\mu} \approx \frac{(1.1 \times 10^{-4} \text{ m})(0.3049 \text{ m/s})(0.7203 \text{ kg/m}^3)}{(2.6 \times 10^{-5} \text{ Pa} \cdot \text{s})}$$

$$\approx 0.93$$

Since the Reynolds number is less than 1, Stokes’ law is valid.