Problem 2B.11

The cone-and-plate viscometer (see Fig. 2B.11). A cone-and-plate viscometer consists of a flat plate and an inverted cone, whose apex just contacts the plate. The liquid whose viscosity is to be measured is placed in the gap between the cone and plate. The cone is rotated at a known angular velocity Ω, and the torque T_z required to turn the cone is measured. Find an expression for the viscosity of the fluid in terms of Ω, T_z, and the angle ψ_0 between the cone and the plate. For commercial instruments ψ_0 is about 1 degree.

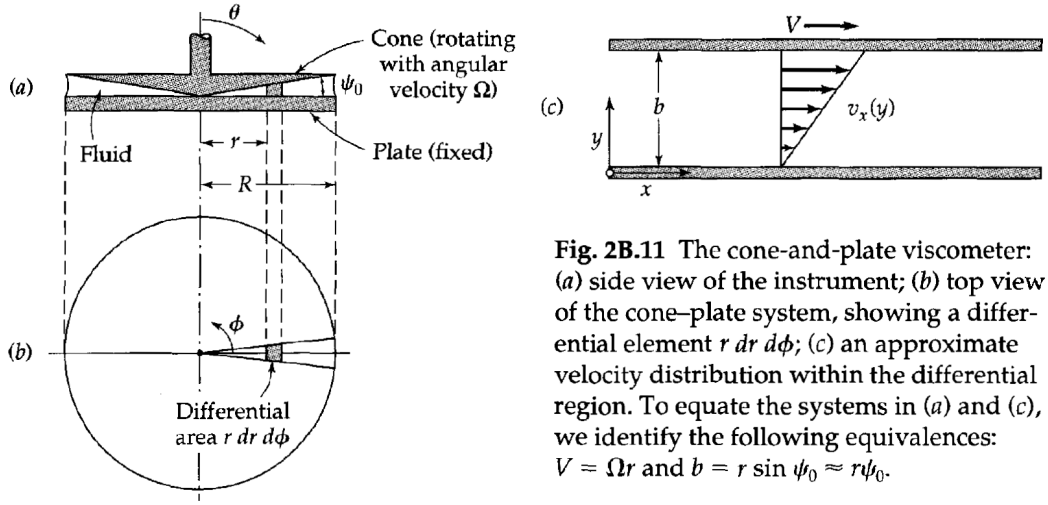

Fig. 2B.11 The cone-and-plate viscometer: (*a*) side view of the instrument; (*b*) top view of the cone–plate system, showing a differential element $r\,dr\,d\phi$; (*c*) an approximate velocity distribution within the differential region. To equate the systems in (*a*) and (*c*), we identify the following equivalences: $V = \Omega r$ and $b = r \sin \psi_0 \approx r\psi_0$.

(a) Assume that locally the velocity distribution in the gap can be very closely approximated by that for flow between parallel plates, the upper one moving with a constant speed. Verify that this leads to the *approximate* velocity distribution (in spherical coordinates)

$$\frac{v_\phi}{r} = \Omega \left(\frac{(\pi/2) - \theta}{\psi_0} \right) \tag{2B.11-1}$$

This approximation should be rather good, because ψ_0 is so small.

(b) From the velocity distribution in Eq. 2B.11-1 and Appendix B.1, show that a reasonable expression for the shear stress is

$$\tau_{\theta\phi} = \mu(\Omega/\psi_0) \tag{2B.11-2}$$

This result shows that the shear stress is uniform throughout the gap. It is this fact that makes the cone-and-plate viscometer quite attractive. The instrument is widely used, particularly in the polymer industry.

(c) Show that the torque required to turn the cone is given by

$$T_z = \frac{2}{3}\pi\mu\Omega R^3/\psi_0 \tag{2B.11-3}$$

This is the standard formula for calculating the viscosity from measurements of the torque and angular velocity for a cone-plate assembly with known R and ψ_0.

(d) For a cone-and-plate instrument with radius 10 cm and angle ψ_0 equal to 0.5 degree, what torque (in dyn · cm) is required to turn the cone at an angular velocity of 10 radians per minute if the fluid viscosity is 100 cp?

Answer: **(d)** $40,000$ dyn · cm