Problem 2B.3

Laminar flow in a narrow slit (see Fig. 2B.3).

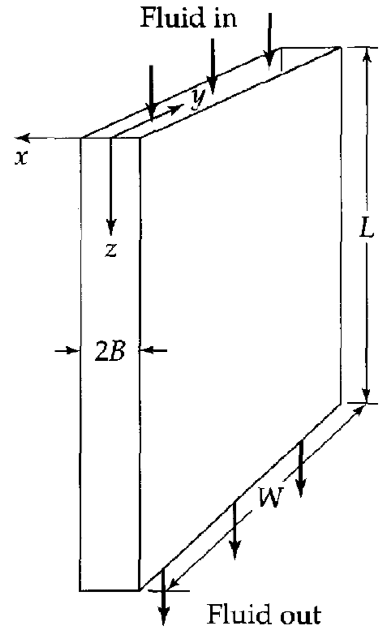

Figure 1: Fig. 2B.3 in the text. Flow through a slit, with $B \ll W \ll L$.

(a) A Newtonian fluid is in laminar flow in a narrow slit formed by two parallel walls a distance $2B$ apart. It is understood that $B \ll W$, so that "edge effects" are unimportant. Make a differential momentum balance, and obtain the following expressions for the momentum-flux and velocity distributions:

$$\tau_{xz} = \left(\frac{\mathscr{P}_0 - \mathscr{P}_L}{L}\right) x \tag{2B.3-1}$$

$$v_z = \frac{(\mathscr{P}_0 - \mathscr{P}_L)B^2}{2\mu L}\left[1 - \left(\frac{x}{B}\right)^2\right] \tag{2B.3-2}$$

In these expressions $\mathscr{P} = p + \rho g h = p - \rho g z$.

(b) What is the ratio of the average velocity to the maximum velocity for this flow?

(c) Obtain the slit analog of the Hagen–Poiseuille equation.

(d) Draw a meaningful sketch to show why the above analysis is inapplicable if $B = W$.

(e) How can the result in (b) be obtained from the results of §2.5?

Answers: **(b)** $\langle v_z \rangle / v_{z,\max} = \dfrac{2}{3}$

(c) $w = \dfrac{2}{3}\dfrac{(\mathscr{P}_0 - \mathscr{P}_L)B^3 W \rho}{\mu L}$