Problem 2B.5

Interrelation of slit and annulus formulas. When an annulus is very thin, it may, to a good approximation, be considered as a thin slit. Then the results of Problem 2B.3 can be taken over with suitable modifications. For example, the mass rate of flow in an annulus with outer wall of radius R and inner wall of radius $(1-\varepsilon)R$, where ε is small, may be obtained from Problem 2B.3 by replacing $2B$ by εR, and W by $2\pi(1-\frac{1}{2}\varepsilon)R$. In this way we get for the mass rate of flow:

$$w = \frac{\pi(\mathscr{P}_0 - \mathscr{P}_L)R^4\varepsilon^3\rho}{6\mu L}\left(1-\frac{1}{2}\varepsilon\right) \tag{2B.5-1}$$

Show that this same result may be obtained from Eq. 2.4-17 by setting κ equal to $1-\varepsilon$ everywhere in the formula and then expanding the expression for w in powers of ε. This requires using the Taylor series (see §C.2)

$$\ln(1-\varepsilon) = -\varepsilon - \frac{1}{2}\varepsilon^2 - \frac{1}{3}\varepsilon^3 - \frac{1}{4}\varepsilon^4 - \cdots \tag{2B.5-2}$$

and then performing a long division. The first term in the resulting series will be Eq. 2B.5-1. *Caution:* In the derivation it is necessary to use the first *four* terms of the Taylor series in Eq. 2B.5-2.