Problem 2B.5

Interrelation of slit and annulus formulas. When an annulus is very thin, it may, to a good approximation, be considered as a thin slit. Then the results of Problem 2B.3 can be taken over with suitable modifications. For example, the mass rate of flow in an annulus with outer wall of radius $R$ and inner wall of radius $(1 - \varepsilon)R$, where $\varepsilon$ is small, may be obtained from Problem 2B.3 by replacing $2B$ by $\varepsilon R$, and $W$ by $2\pi(1 - \frac{1}{2}\varepsilon)R$. In this way we get for the mass rate of flow:

$$w = \frac{\pi(P_0 - P_L)R^4\varepsilon^3\rho}{6\mu L} \left(1 - \frac{1}{2}\varepsilon\right)$$  \hspace{1cm} (2B.5-1)

Show that this same result may be obtained from Eq. 2.4-17 by setting $\kappa$ equal to $1 - \varepsilon$ everywhere in the formula and then expanding the expression for $w$ in powers of $\varepsilon$. This requires using the Taylor series (see §C.2)

$$\ln(1 - \varepsilon) = -\varepsilon - \frac{1}{2}\varepsilon^2 - \frac{1}{3}\varepsilon^3 - \frac{1}{4}\varepsilon^4 - \cdots$$  \hspace{1cm} (2B.5-2)

and then performing a long division. The first term in the resulting series will be Eq. 2B.5-1. **Caution:** In the derivation it is necessary to use the first four terms of the Taylor series in Eq. 2B.5-2.