Problem 2B.6

Flow of a film on the outside of a circular tube (see Fig. 2B.6). In a gas absorption experiment a viscous fluid flows upward through a small circular tube and then downward in laminar flow on the outside. Set up a momentum balance over a shell of thickness Δr in the film, as shown in Fig. 2B.6. Note that the "momentum in" and "momentum out" arrows are always taken in the positive coordinate direction, even though in this problem the momentum is flowing through the cylindrical surfaces in the negative r direction.

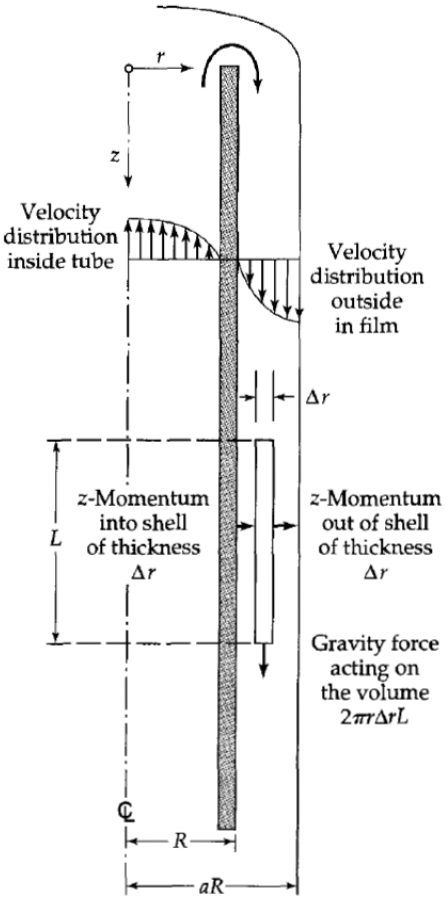

Figure 1: Fig. 2B.6 in the text. Velocity distribution and z-momentum balance for the flow of a falling film on the outside of a circular tube.

(a) Show that the velocity distribution in the falling film (neglecting end effects) is

$$v_z = \frac{\rho g R^2}{4\mu}\left[1 - \left(\frac{r}{R}\right)^2 + 2a^2 \ln\left(\frac{r}{R}\right)\right] \tag{2B.6-1}$$

(b) Obtain an expression for the mass rate of flow in the film.

(c) Show that the result in (b) simplifies to Eq. 2.2-21 if the film thickness is very small.