Problem 2B.7

Annular flow with inner cylinder moving axially (see Fig. 2B.7). A cylindrical rod of radius κR moves axially with velocity $v_z = v_0$ along the axis of a cylindrical cavity of radius R as seen in the figure. The pressure at both ends of the cavity is the same, so that the fluid moves through the annular region solely because of the rod motion.

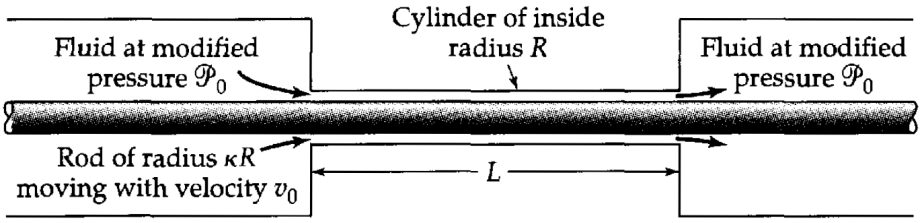

Fig. 2B.7 Annular flow with the inner cylinder moving axially.

(a) Find the velocity distribution in the narrow annular region.

(b) Find the mass rate of flow through the annular region.

(c) Obtain the viscous force acting on the rod over the length L.

(d) Show that the result in (c) can be written as a "plane slit" formula multiplied by a "curvature correction." Problems of this kind arise in studying the performance of wire-coating dies.[1]

Answers: **(a)** $\dfrac{v_z}{v_0} = \dfrac{\ln(r/R)}{\ln \kappa}$

(b) $w = \dfrac{\pi R^2 v_0 \rho}{2}\left[\dfrac{(1-\kappa^2)}{\ln(1/\kappa)} - 2\kappa^2\right]$

(c) $F_z = -2\pi L \mu v_0 / \ln(1/\kappa)$

(d) $F_z = \dfrac{-2\pi L \mu v_0}{\varepsilon}\left(1 - \dfrac{1}{2}\varepsilon - \dfrac{1}{12}\varepsilon^2 + \cdots\right)$ where $\varepsilon = 1 - \kappa$ (see Problem 2B.5)

[1] J. B. Paton, P. H. Squires, W. H. Darnell, F. M. Cash, and J. F. Carley, *Processing of Thermoplastic Materials*, E. C. Bernhardt (ed.), Reinhold, New York (1959), Chapter 4.