Problem 2B.9

Low-density phenomena in compressible tube flow[2,3] (see Fig. 2B.9). As the pressure is decreased in the system studied in Example 2.3-2, deviations from Eqs. 2.3-28 and 2.3-29 arise. The gas behaves as if it slips at the tube wall. It is conventional[2] to replace the customary "no-slip" boundary condition that $v_z = 0$ at the tube wall by

$$v_z = -\zeta \frac{dv_z}{dr}, \qquad \text{at } r = R \tag{2B.9-1}$$

in which ζ is the *slip coefficient*. Repeat the derivation in Example 2.3-2 using Eq. 2B.9-1 as the boundary condition. Also make use of the experimental fact that the slip coefficient varies inversely with the pressure $\zeta = \zeta_0/p$, in which ζ_0 is a constant. Show that the mass rate of flow is

$$w = \frac{\pi(p_0 - p_L)R^4 \rho_{\text{avg}}}{8\mu L}\left(1 + \frac{4\zeta_0}{R p_{\text{avg}}}\right) \tag{2B.9-2}$$

in which $p_{\text{avg}} = \frac{1}{2}(p_0 + p_L)$ and ρ_{avg} is the average density calculated at p_{avg}.

When the pressure is decreased further, a flow regime is reached in which the mean free path of the gas molecules is large with respect to the tube radius (*Knudsen flow*). In that regime[3]

$$w = \sqrt{\frac{2m}{\pi \kappa T}}\left(\frac{4}{3}\pi R^3\right)\left(\frac{p_0 - p_L}{L}\right) \tag{2B.9-3}$$

in which m is the molecular mass and κ is the Boltzmann constant. In the derivation of this result it is assumed that all collisions of the molecules with the solid surfaces are *diffuse* and not *specular*. The results in Eqs. 2.3-29, 2B.9-2, and 2B.9-3 are summarized in Fig. 2B.9.

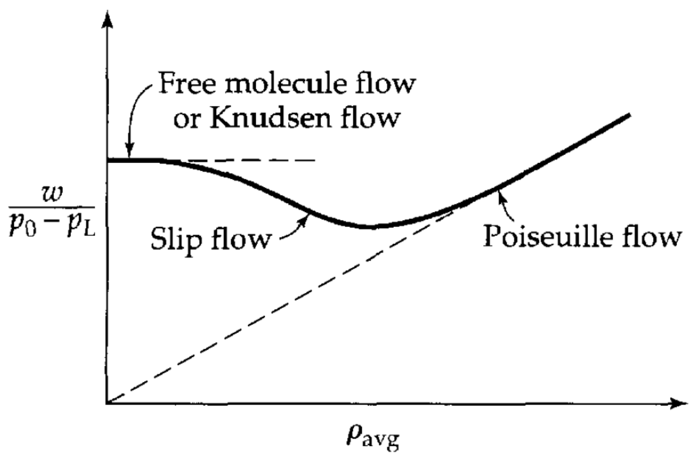

Fig. 2B.9 A comparison of the flow regimes in gas flow through a tube.

[2] E. H. Kennard, *Kinetic Theory of Gases*, McGraw-Hill, New York (1938), pp. 292-295, 300-306.

[3] M. Knudsen, *The Kinetic Theory of Gases*, Methuen, London, 3rd edition (1950). See also R. J. Silbey and R. A. Alberty, *Physical Chemistry*, Wiley, New York, 3rd edition (2001), §17.6.