Problem 2C.4

Falling-cylinder viscometer (see Fig. 2C.4).[6] A falling-cylinder viscometer consists of a long vertical cylindrical container (radius R) capped at both ends, with a solid cylindrical slug (radius κR). The slug is equipped with fins so that its axis is coincident with that of the tube.

One can observe the rate of descent of the slug in the cylindrical container when the latter is filled with fluid. Find an equation that gives the viscosity of the fluid in terms of the terminal velocity v_0 of the slug and the various geometrical quantities shown in the figure.

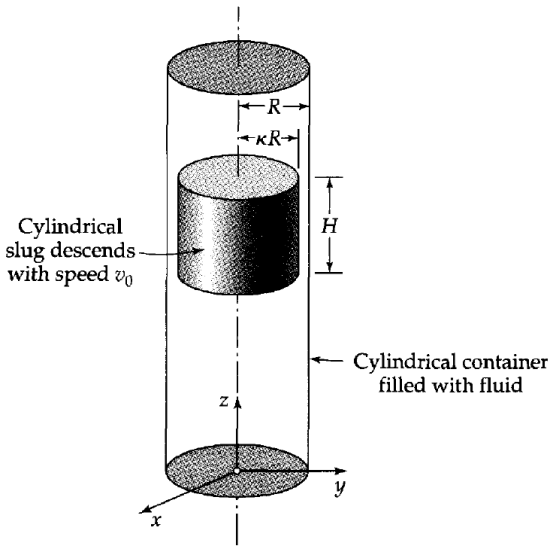

Fig. 2C.4 A falling-cylinder viscometer with a tightly fitting solid cylinder moving vertically. The cylinder is usually equipped with fins to maintain centering within the tube. The fluid completely fills the tube, and the top and bottom are closed.

(a) Show that the velocity distribution in the annular slit is given by

$$\frac{v_z}{v_0} = -\frac{(1-\xi^2)-(1+\kappa^2)\ln(1/\xi)}{(1-\kappa^2)-(1+\kappa^2)\ln(1/\kappa)} \tag{2C.4-1}$$

in which $\xi = r/R$ is a dimensionless radial coordinate.

(b) Make a force balance on the cylindrical slug and obtain

$$\mu = \frac{(\rho_0-\rho)g(\kappa R)^2}{2v_0}\left[\left(\ln\frac{1}{\kappa}\right)-\left(\frac{1-\kappa^2}{1+\kappa^2}\right)\right] \tag{2C.4-2}$$

in which ρ and ρ_0 are the densities of the fluid and the slug, respectively.

(c) Show that, for small slit widths, the result in (b) may be expanded in powers of $\varepsilon = 1-\kappa$ to give

$$\mu = \frac{(\rho_0-\rho)gR^2\varepsilon^3}{6v_0}\left(1-\frac{1}{2}\varepsilon-\frac{13}{20}\varepsilon^2+\cdots\right) \tag{2C.4-3}$$

See §C.2 for information on expansions in Taylor series.

[6] J. Lohrenz, G. W. Swift, and F. Kurata, *AIChE Journal*, **6**, 547-550 (1960) and **7**, 6S (1961); E. Ashare, R. B. Bird, and J. A. Lescarboura, *AIChE Journal*, **11**, 910-916 (1965) F. J. Eichstadt and G. W. Swift, *AIChE Journal*, **12**, 1179-1183 (1966); M. C. S. Chen, J. A. Lescarboura, *AIChE Journal*, **14**, 123-127 (1968).