Problem 2C.5

Falling film on a conical surface (see Fig. 2C.5).[7] A fluid flows upward through a circular tube and then downward on a conical surface. Find the film thickness as a function of the distance s down the cone.

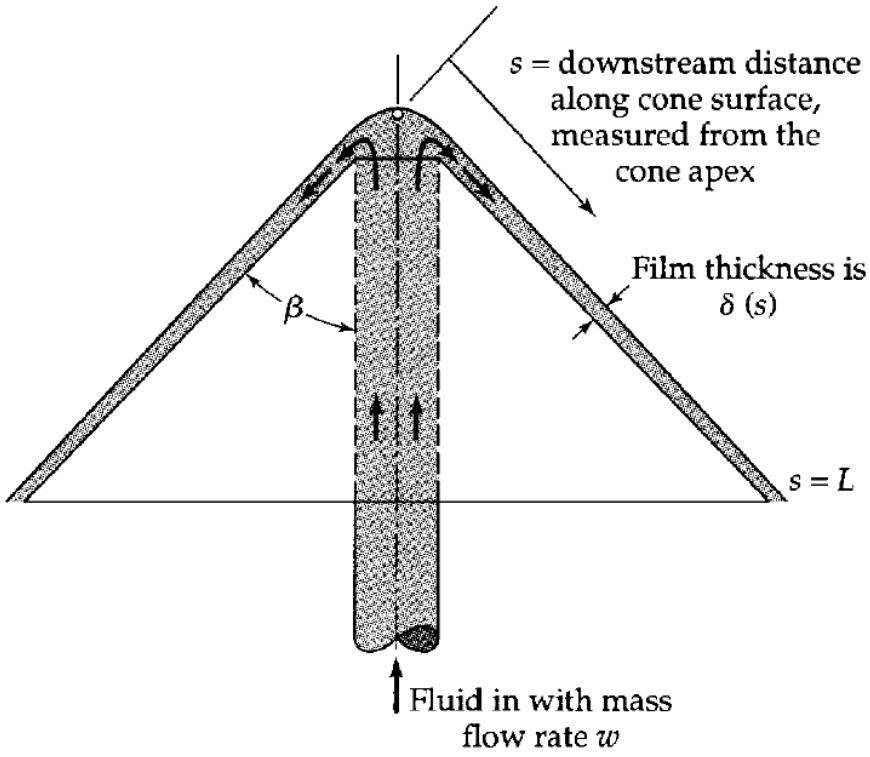

Fig. 2C.5 A falling film on a conical surface.

(a) Assume that the results of §2.2 apply *approximately* over any small region of the cone surface. Show that a mass balance on a ring of liquid contained between s and $s + \Delta s$ gives:

$$\frac{d}{ds}(s\delta\langle v\rangle) = 0 \quad \text{or} \quad \frac{d}{ds}(s\delta^3) = 0 \tag{2C.5-1}$$

(b) Integrate this equation and evaluate the constant of integration by equating the mass rate of flow w up the central tube to that flowing down the conical surface at $s = L$. Obtain the following expression for the film thickness:

$$\delta = \sqrt[3]{\frac{3\mu w}{\pi\rho^2 g L \sin 2\beta}\left(\frac{L}{s}\right)} \tag{2C.5-2}$$

[7] R. B. Bird, in *Selected Topics in Transport Phenomena*, CEP Symposium Series #58, **61**, 1-15 (1965).