Problem 2C.7

A simple rate-of-climb indicator (see Fig. 2C.7). Under the proper circumstances the simple apparatus shown in the figure can be used to measure the rate of climb of an airplane. The gauge pressure inside the Bourdon element is taken as proportional to the rate of climb. For the purposes of this problem the apparatus may be assumed to have the following properties: (i) the capillary tube (of radius R and length L, with $R \ll L$) is of negligible volume but appreciable flow resistance; (ii) the Bourdon element has a constant volume V and offers negligible resistance to the flow; and (iii) flow in the capillary is laminar and incompressible, and the volumetric flow rate depends only on the conditions at the ends of the capillary.

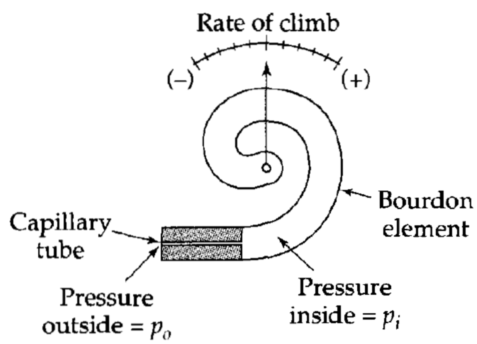

Fig. 2C.7 A rate-of-climb indicator.

(a) Develop an expression for the change of pressure with altitude, neglecting temperature changes, and considering air to be an ideal gas of constant composition. (*Hint:* Write a shell balance in which the weight of gas is balanced against the static pressure.)

(b) By making a mass balance over the gauge, develop an approximate relation between gauge pressure $p_i - p_o$ and rate of climb v_z for a long continued constant-rate climb. Neglect change of air viscosity, and assume changes in air density to be small.

(c) Develop an approximate expression for the "relaxation time" t_{rel} of the indicator—that is, the time required for the gauge pressure to drop to $1/e$ of its initial value when the external pressure is suddenly changed from zero (relative to the interior of the gauge) to some different constant value, and maintained indefinitely at this new value.

(d) Discuss the usefulness of this type of indicator for small aircraft.

(e) Justify the plus and minus signs in the figure.

Answers: **(a)** $dp/dz = -\rho g = -(pM/RT)g$

(b) $p_i - p_o \approx v_z(8\mu L/\pi R^4)(MgV/R_gT)$, where R_g is the gas constant and M is the molecular weight.

(c) $t_{\text{rel}} = (128/\pi)(\mu VL/D^4\bar{p})$, where $\bar{p} = \dfrac{1}{2}(p_i + p_o)$