Problem 2D.2

Drainage of liquids[9] (see Fig. 2D.2). How much liquid clings to the inside surface of a large vessel when it is drained? As shown in the figure there is a thin film of liquid left behind on the wall as the liquid level in the vessel falls. The local film thickness is a function of both z (the distance down from the initial liquid level) and t (the elapsed time).

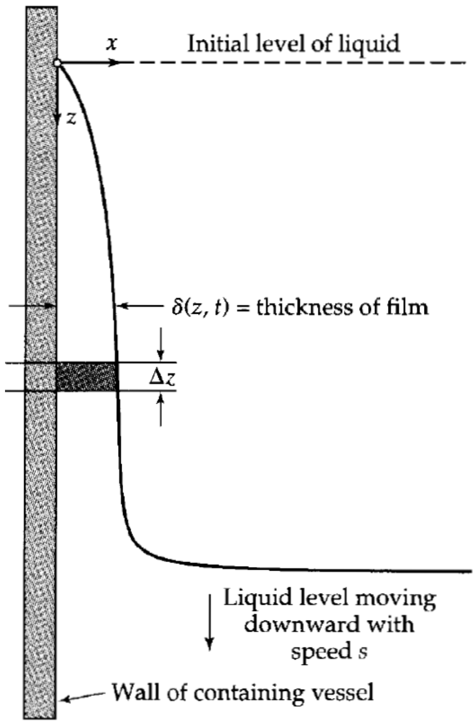

Fig. 2D.2 Clinging of a viscous fluid to wall of vessel during draining.

(a) Make an unsteady-state mass balance on a portion of the film between z and $z + \Delta z$ to get

$$\frac{\partial}{\partial z}\langle v_z \rangle \delta = -\frac{\partial \delta}{\partial t} \tag{2D.2-1}$$

(b) Use Eq. 2.2-18 and a quasi-steady-assumption to obtain the following first-order partial differential equation for $\delta(z, t)$:

$$\frac{\partial \delta}{\partial t} + \frac{\rho g}{\mu}\delta^2 \frac{\partial \delta}{\partial z} = 0 \tag{2D.2-2}$$

(c) Solve this equation to get

$$\delta(z, t) = \sqrt{\frac{\mu}{\rho g}\frac{z}{t}} \tag{2D.2-3}$$

What restrictions have to be placed on this result?

[9] J. J. van Rossum, *Appl. Sci. Research*, **47**, 121-144 (1958); see also V. G. Levich, *Physicochemical Hydrodynamics*, Prentice-Hall, Englewood Cliffs, N.J. (1962), Chapter 12.