Problem 3A.5

Fabrication of a parabolic mirror. It is proposed to make a backing for a parabolic mirror, by rotating a pan of slow-hardening plastic resin at constant speed until it hardens (see Fig. 3.6-6). Calculate the rotational speed required to produce a mirror of focal length $f = 100$ cm. The focal length is one-half the radius of curvature at the axis, which in turn is given by

$$r_c = \left[1 + \left(\frac{dz}{dr}\right)^2\right]^{3/2} \left(\frac{d^2z}{dr^2}\right)^{-1} \tag{3A.5-1}$$

Answer: 21.1 rpm

Solution

According to Example 3.6-4 on pages 93–95, the shape of a liquid in a rotating circular cylinder is a parabola.

$$z - z_0 = \left(\frac{\Omega^2}{2g}\right) r^2$$

Differentiate both sides with respect to r twice to get dz/dr and d^2z/dr^2.

$$z - z_0 = \left(\frac{\Omega^2}{2g}\right) r^2 \quad \rightarrow \quad \frac{dz}{dr} = \frac{\Omega^2}{g} r \quad \rightarrow \quad \frac{d^2z}{dr^2} = \frac{\Omega^2}{g}$$

Plug these results into the provided formula for the radius of curvature.

$$r_c = \left[1 + \left(\frac{\Omega^2}{g} r\right)^2\right]^{3/2} \left(\frac{\Omega^2}{g}\right)^{-1}$$

Note that at the axis of rotation $r = 0$.

$$r_c = \left(\frac{\Omega^2}{g}\right)^{-1}$$

Since we want a focal length of $f = 100$ cm, the radius of curvature at the axis will be $r_c = 200 \text{ cm} = 2$ m.

$$2 = \left(\frac{\Omega^2}{g}\right)^{-1}$$

Solve for the angular velocity Ω and convert it to the desired units of revolutions per minute.

$$\begin{aligned} \Omega &= \sqrt{\frac{g}{2}} \\ &\approx 2.21 \frac{\cancel{\text{rad}}}{\cancel{\text{s}}} \times \frac{1 \text{ rev}}{2\pi \cancel{\text{rad}}} \times \frac{60 \cancel{\text{s}}}{1 \text{ min}} \\ &\approx 21.1 \text{ rpm} \end{aligned}$$