Problem 3B.14

Torricelli's equation for efflux from a tank (Fig. 3B.14). A large uncovered tank is filled with a liquid to a height h. Near the bottom of the tank, there is a hole that allows the fluid to exit to the atmosphere. Apply Bernoulli's equation to a streamline that extends from the surface of the liquid at the top to a point in the exit stream just outside the vessel. Show that this leads to an efflux velocity $v_{\text{efflux}} = \sqrt{2gh}$. This is known as *Torricelli's equation.*

To get this result, one has to assume incompressibility (which is usually reasonable for most liquids), and that the height of the fluid surface is changing so slowly with time that the Bernoulli equation can be applied at any instant of time (the quasi-steady-state assumption).

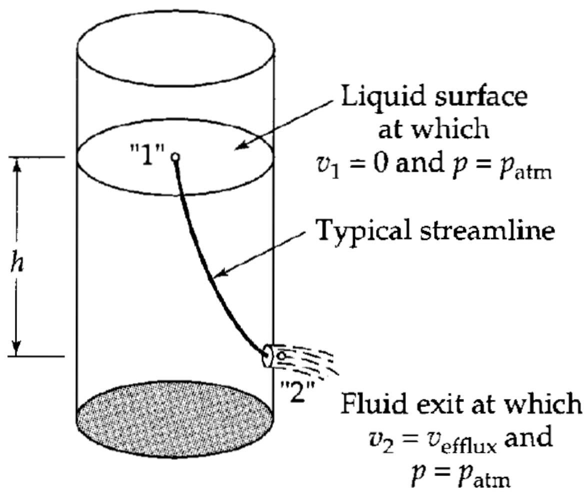

Fig. 3B.14. Fluid draining from a tank. Points "1" and "2" are on the same streamline.

Solution

Bernoulli's equation is valid for steady flow of an inviscid fluid. For two points along a streamline in the flow, the equation relates the velocities, pressures, and heights.

$$\frac{1}{2}(v_2^2 - v_1^2) + \int_{p_1}^{p_2} \frac{1}{\rho}\, dp + g(h_2 - h_1) = 0$$

The liquid is assumed to be incompressible, so the density is independent of pressure.

$$\frac{1}{2}(v_2^2 - v_1^2) + \frac{1}{\rho}(p_2 - p_1) + g(h_2 - h_1) = 0$$

The liquid surface and the hole are exposed to the atmosphere, so the pressure at these points is p_{atm}. From Fig. 3B.14, we can see that $h_1 - h_2 = h$. The surface is assumed to be stationary $v_1 = 0$, and the exit velocity $v_2 = v_{\text{efflux}}$ is unknown. The Bernoulli equation becomes

$$\frac{1}{2}v_{\text{efflux}}^2 + g(-h) = 0.$$

Therefore,

$$v_{\text{efflux}} = \sqrt{2gh}.$$