Problem 3B.10

Radial flow between parallel disks (Fig. 3B.10). A part of a lubrication system consists of two circular disks between which a lubricant flows radially. The flow takes place because of a modified pressure difference $\mathscr{P}_1 - \mathscr{P}_2$ between the inner and outer radii r_1 and r_2, respectively.

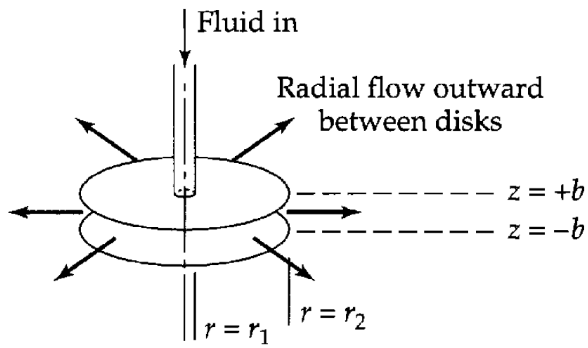

Fig. 3B.10. Outward radial flow in the space between two parallel, circular disks.

(a) Write the equations of continuity and motion for this flow system, assuming steady-state, laminar, incompressible Newtonian flow. Consider only the region $r_1 \le r \le r_2$ and a flow that is radially directed.

(b) Show how the equation of continuity enables one to simplify the equation of motion to give

$$-\rho \frac{\phi^2}{r^3} = -\frac{d\mathscr{P}}{dr} + \mu \frac{1}{r}\frac{d^2\phi}{dz^2} \tag{3B.10-1}$$

in which $\phi = r v_r$ is a function of z only. Why is ϕ independent of r?

(c) It can be shown that no solution exists for Eq. 3B.10-1 unless the nonlinear term containing ϕ is omitted. Omission of this term corresponds to the "creeping flow assumption." Show that for creeping flow, Eq. 3B.10-1 can be integrated with respect to r to give

$$0 = (\mathscr{P}_1 - \mathscr{P}_2) + \left(\mu \ln \frac{r_2}{r_1}\right) \frac{d^2\phi}{dz^2} \tag{3B.10-2}$$

(d) Show that further integration with respect to z gives

$$v_r(r,z) = \frac{(\mathscr{P}_1 - \mathscr{P}_2) b^2}{2\mu r \ln(r_2/r_1)} \left[1 - \left(\frac{z}{b}\right)^2\right] \tag{3B.10-3}$$

(e) Show that the mass flow rate is

$$w = \frac{4\pi(\mathscr{P}_1 - \mathscr{P}_2) b^3 \rho}{3\mu \ln(r_2/r_1)} \tag{3B.10-4}$$

(f) Sketch the curves $\mathscr{P}(r)$ and $v_r(r,z)$.