

Problem 3B.12

Pressure distribution in incompressible fluids. Penelope is staring at a beaker filled with a liquid, which for all practical purposes can be considered as incompressible; let its density be ρ_0 . She tells you she is trying to understand how the pressure in the liquid varies with depth. She has taken the origin of coordinates at the liquid–air interface, with the positive z -axis pointing away from the liquid. She says to you:

“If I simplify the equation of motion for an incompressible liquid at rest, I get $0 = -dp/dz - \rho_0 g$. I can solve this and get $p = p_{\text{atm}} - \rho_0 g z$. That seems reasonable—the pressure increases with increasing depth.

“But, on the other hand, the equation of state for any fluid is $p = p(\rho, T)$, and if the system is at constant temperature, this just simplifies to $p = p(\rho)$. And, since the fluid is incompressible, $p = p(\rho_0)$, and p must be a constant throughout the fluid! How can that be?”

Clearly Penelope needs help. Provide a useful explanation.