

Problem 3B.2

Laminar flow in a triangular duct (Fig. 3B.2).² One type of compact heat exchanger is shown in Fig. 3B.2(a). In order to analyze the performance of such an apparatus, it is necessary to understand the flow in a duct whose cross section is an equilateral triangle. This is done most easily by installing a coordinate system as shown in Fig. 3B.2(b).

- (a) Verify that the velocity distribution for the laminar flow of a Newtonian fluid in a duct of this type is given by

$$v_z = \frac{(\mathcal{P}_0 - \mathcal{P}_L)}{4\mu LH}(y - H)(3x^2 - y^2) \quad (3B.2-1)$$

- (b) From Eq. 3B.2-1 find the average velocity, maximum velocity, and mass flow rate.

$$\text{Answers: (b) } \langle v_z \rangle = \frac{(\mathcal{P}_0 - \mathcal{P}_L)H^2}{60\mu L} = \frac{9}{20}v_{z,\max};$$

$$w = \frac{\sqrt{3}(\mathcal{P}_0 - \mathcal{P}_L)H^4\rho}{180\mu L}$$

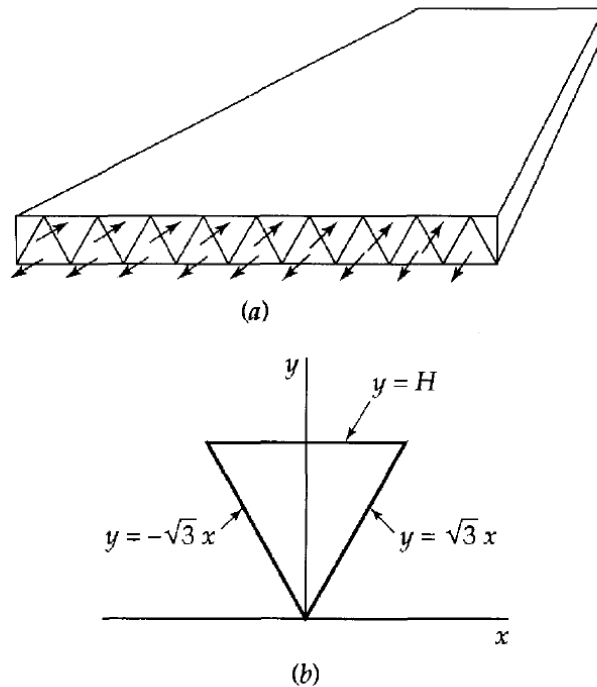


Fig. 3B.2. (a) Compact heat-exchanger element, showing channels of a triangular cross section; (b) coordinate system for an equilateral-triangular duct.

²An alternative formulation of the velocity profile is given by L. D. Landau and E. M. Lifshitz, *Fluid Mechanics*, Pergamon, Oxford, 2nd edition (1987), p. 54.