

Problem 3B.3

Laminar flow in a square duct.

- (a) A straight duct extends in the z direction for a length L and has a square cross section, bordered by the lines $x = \pm B$ and $y = \pm B$. A colleague has told you that the velocity distribution is given by

$$v_z = \frac{(\mathcal{P}_0 - \mathcal{P}_L)B^2}{4\mu L} \left[1 - \left(\frac{x}{B} \right)^2 \right] \left[1 - \left(\frac{y}{B} \right)^2 \right] \quad (3B.3-1)$$

Since this colleague has occasionally given you wrong advice in the past, you feel obliged to check the result. Does it satisfy the relevant boundary conditions and the relevant differential equation?

- (b) According to the review article by Berker,³ the mass rate of flow in a square duct is given by

$$w = \frac{0.563(\mathcal{P}_0 - \mathcal{P}_L)B^4\rho}{\mu L} \quad (3B.3-2)$$

Compare the coefficient in this expression with the coefficient that one obtains from Eq. 3B.3-1.

³R. Berker, *Handbuch der Physik*, Vol. VIII/2, Springer, Berlin (1963); see pp. 67–77 for laminar flow in conduits of noncircular cross sections. See also W. E. Stewart, *AIChE Journal*, **8**, 425–428 (1962).