Problem 3B.4

Creeping flow between two concentric spheres (Fig. 3B.4). A very viscous Newtonian fluid flows in the space between two concentric spheres, as shown in the figure. It is desired to find the rate of flow in the system as a function of the imposed pressure difference. Neglect end effects and postulate that v_θ depends only on r and θ with the other velocity components zero.

(a) Using the equation of continuity, show that $v_\theta \sin\theta = u(r)$, where $u(r)$ is a function of r to be determined.

(b) Write the θ-component of the equation of motion for this system, assuming the flow to be slow enough that the $[\mathbf{v} \cdot \nabla \mathbf{v}]$ term is negligible. Show that this gives

$$0 = -\frac{1}{r}\frac{\partial \mathscr{P}}{\partial \theta} + \mu \left[\frac{1}{\sin\theta}\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{du}{dr}\right)\right] \tag{3B.4-1}$$

(c) Separate this into two equations

$$\sin\theta \frac{\partial \mathscr{P}}{\partial \theta} = B; \qquad \frac{\mu}{r}\frac{d}{dr}\left(r^2 \frac{du}{dr}\right) = B \tag{3B.4-2, 3}$$

where B is the separation constant, and solve the two equations to get

$$B = \frac{\mathscr{P}_2 - \mathscr{P}_1}{2 \ln \cot \frac{1}{2}\varepsilon} \tag{3B.4-4}$$

$$u(r) = \frac{(\mathscr{P}_1 - \mathscr{P}_2)R}{4\mu \ln \cot(\varepsilon/2)}\left[\left(1 - \frac{r}{R}\right) + \kappa\left(1 - \frac{R}{r}\right)\right] \tag{3B.4-5}$$

where $\mathscr{P}_1$ and $\mathscr{P}_2$ are the values of the modified pressure at $\theta = \varepsilon$ and $\theta = \pi - \varepsilon$, respectively.

(d) Use the results above to get the mass rate of flow

$$w = \frac{\pi(\mathscr{P}_1 - \mathscr{P}_2)R^3(1-\kappa)^3\rho}{12\mu \ln \cot(\varepsilon/2)} \tag{3B.4-6}$$

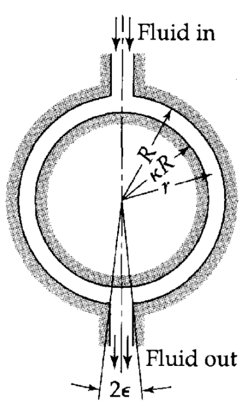

Fig. 3B.4. Creeping flow in the region between two stationary concentric spheres.