Problem 3B.5

Parallel-disk viscometer (Fig. 3B.5). A fluid, whose viscosity is to be measured, is placed in the gap of thickness B between the two disks of radius R. One measures the torque T_z required to turn the upper disk at an angular velocity Ω. Develop the formula for deducing the viscosity from these measurements. Assume creeping flow.

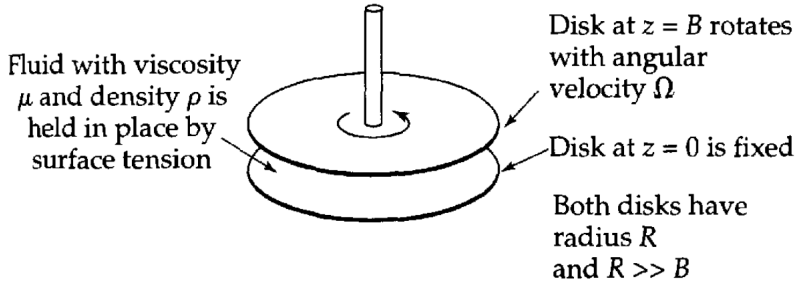

Fig. 3B.5. Parallel-disk viscometer.

(a) Postulate that for small values of Ω the velocity profiles have the form $v_r = 0$, $v_z = 0$, and $v_\theta = rf(z)$; why does this form for the tangential velocity seem reasonable? Postulate further that $\mathscr{P} = \mathscr{P}(r, z)$. Write down the resulting simplified equations of continuity and motion.

(b) From the θ-component of the equation of motion, obtain a differential equation for $f(z)$. Solve the equation for $f(z)$ and evaluate the constants of integration. This leads ultimately to the result $v_\theta = \Omega r(z/B)$. Could you have guessed this result?

(c) Show that the desired working equation for deducing the viscosity is $\mu = 2BT_z/\pi\Omega R^4$.

(d) Discuss the advantages and disadvantages of this instrument.