Problem 3C.3

Deformation of a fluid line (Fig. 3C.3). A fluid is contained in the annular space between two cylinders of radii κR and R. The inner cylinder is made to rotate with a constant angular velocity of Ω_i. Consider a line of fluid particles in the plane $z = 0$ extending from the inner cylinder to the outer cylinder and initially located at $\theta = 0$, normal to the two surfaces. How does this fluid line deform into a curve $\theta(r,t)$? What is the length, l, of the curve after N revolutions of the inner cylinder? Use Eq. 3.6-32.

$$\textit{Answer: } \frac{l}{R} = \int_\kappa^1 \sqrt{1 + \frac{16\pi^2 N^2}{[(1/\kappa)^2 - 1]^2 \xi^4}}\, d\xi$$

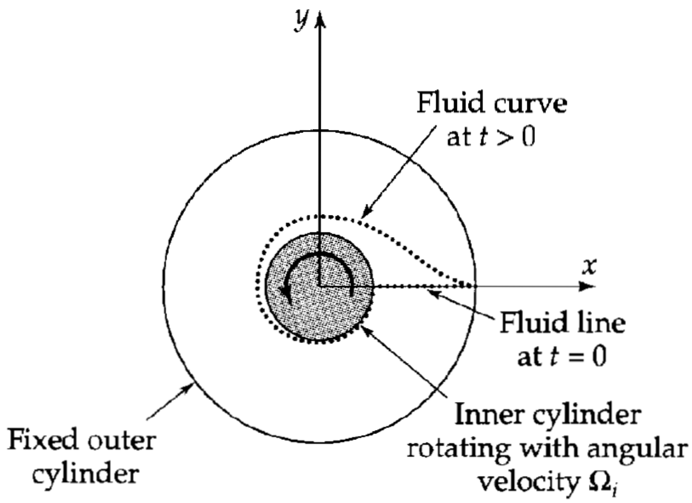

Fig. 3C.3. Deformation of a fluid line in Couette flow.