Problem 4A.3

Construction of streamlines for the potential flow around a cylinder. Plot the streamlines for the flow around a cylinder using the information in Example 4.3-1 by the following procedure:

(a) Select a value of $\Psi = C$ (that is, select a streamline).

(b) Plot $Y = C + K$ (straight lines parallel to the X-axis) and $Y = K(X^2 + Y^2)$ (circles with radius $1/2K$, tangent to the X-axis at the origin).

(c) Plot the intersections of the lines and circles that have the same value of K.

(d) Join these points to get the streamline for $\Psi = C$.

Then select other values of C and repeat the process until the pattern of streamlines is clear.

[TYPOS: Plot $Y = K - C$ instead. Also, the radius of the circle should be $1/2|K|$.]

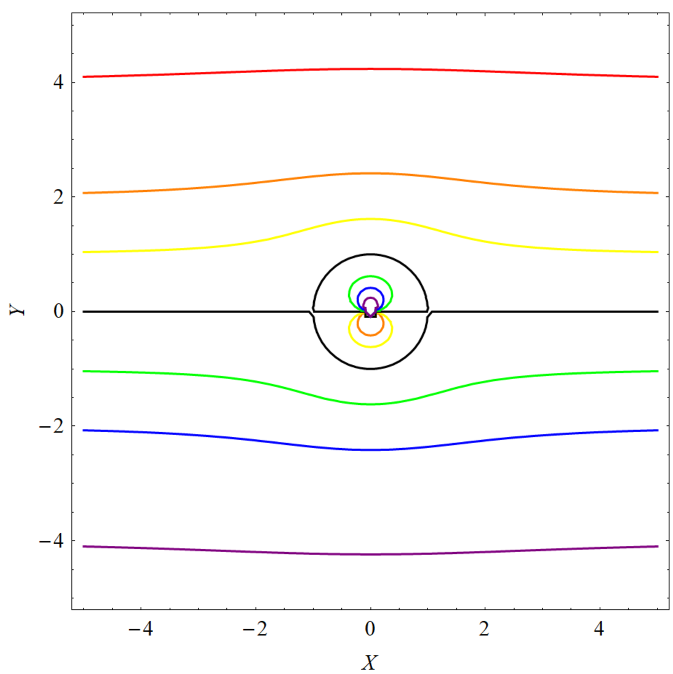
Solution

According to Example 4.3-1, the stream function in terms of the dimensionless variables, Ψ and X and Y, is given by Eq. 4.3-19.

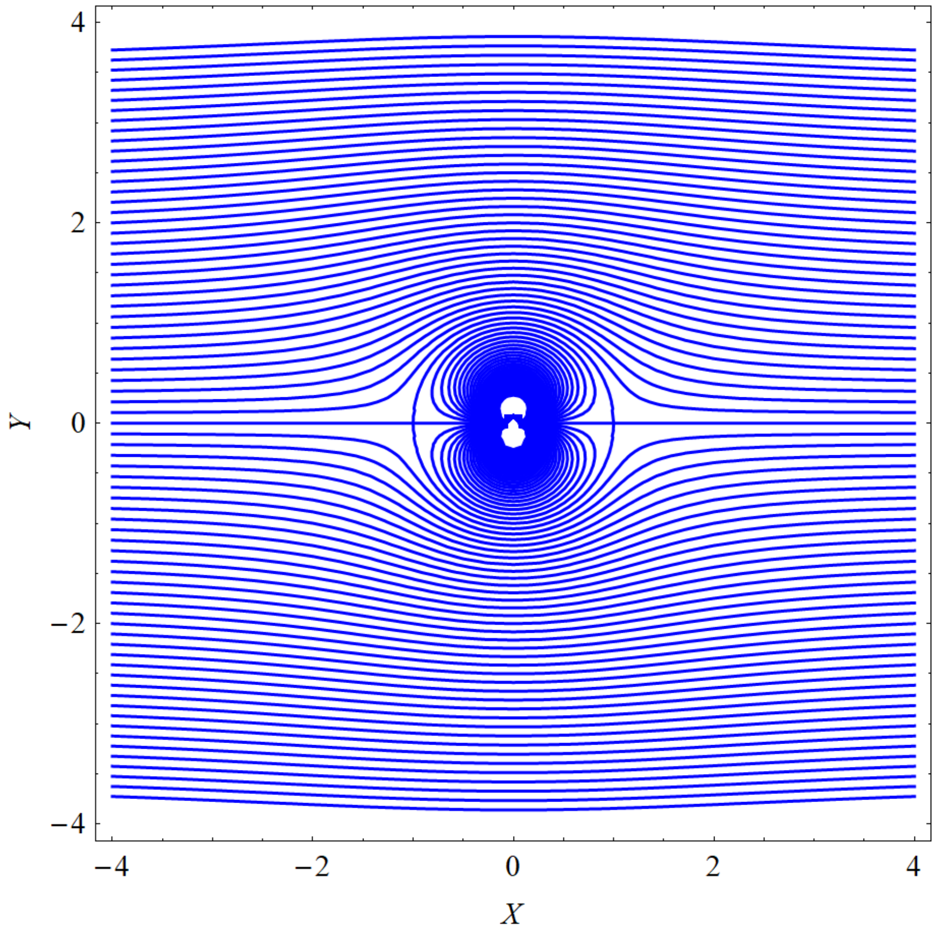
$$\Psi(X, Y) = -Y\left(1 - \frac{1}{X^2 + Y^2}\right) \tag{4.3-19}$$

With a Computer

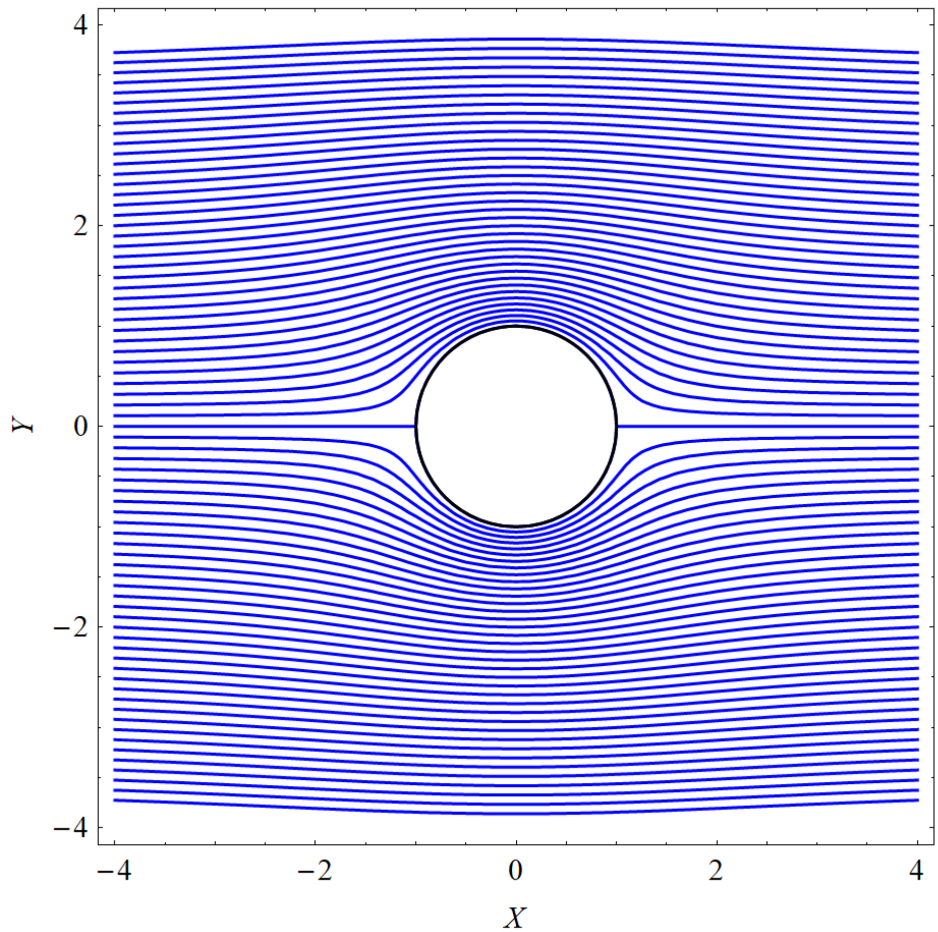
Below is a graph of $\Psi(X, Y) = -4$, $\Psi(X, Y) = -2$, $\Psi(X, Y) = -1$, $\Psi(X, Y) = 1$, $\Psi(X, Y) = 2$, $\Psi(X, Y) = 4$, and $\Psi(X, Y) = 0$ in red, orange, yellow, green, blue, purple, and black, respectively.



Decrease the spacing between streamlines by plotting $\Psi(X, Y) = -4$, $\Psi(X, Y) = -3.9$, $\Psi(X, Y) = -3.8$, ..., $\Psi(X, Y) = 3.8$, $\Psi(X, Y) = 3.9$, and $\Psi(X, Y) = 4$.



Highlight the circle $X^2 + Y^2 = 1$ black and ignore the ink within it to obtain a picture of the fluid flow around a cylinder.



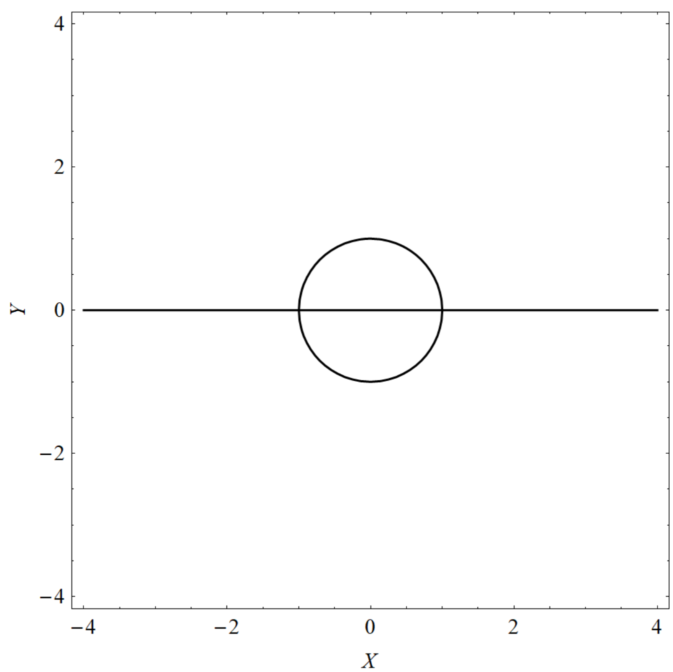
Without a Computer

Pretend that we don't have software sophisticated enough to graph the streamlines $\Psi(X, Y) = C$. Then we have to tediously construct them one by one.

$$\Psi(X, Y) = -Y\left(1 - \frac{1}{X^2 + Y^2}\right) = C$$

The issue is that this is a cubic equation for Y, so the aim is to break it down into simpler functions that we can graph. Consider the case that $C = 0$. Then by the zero product theorem,

$$\begin{aligned} -Y &= 0 \qquad \text{or} \qquad 1 - \frac{1}{X^2 + Y^2} = 0 \\ Y &= 0 \qquad \text{or} \qquad X^2 + Y^2 = 1. \end{aligned}$$



Now consider the case that C is nonzero.

$$\Psi(X, Y) = -Y + \frac{Y}{X^2 + Y^2} = C$$

$$\frac{Y}{X^2 + Y^2} = C + Y$$

The only way a function of X and Y can be equal to a function of Y is if both are equal to some constant K.

$$\frac{Y}{X^2 + Y^2} = C + Y = K \quad \Rightarrow \quad \begin{cases} C + Y = K \\ \dfrac{Y}{X^2 + Y^2} = K \end{cases}$$

The two simpler functions we have to graph then are

$$\begin{aligned} Y &= K - C \\ Y &= K(X^2 + Y^2). \end{aligned}$$

This second function can actually be written as

$$X^2 + Y^2 - \frac{Y}{K} = 0$$
$$X^2 + Y^2 - \frac{Y}{K} + \frac{1}{4K^2} = \frac{1}{4K^2}$$
$$X^2 + \left(Y - \frac{1}{2K}\right)^2 = \frac{1}{4K^2},$$

which is a circle with

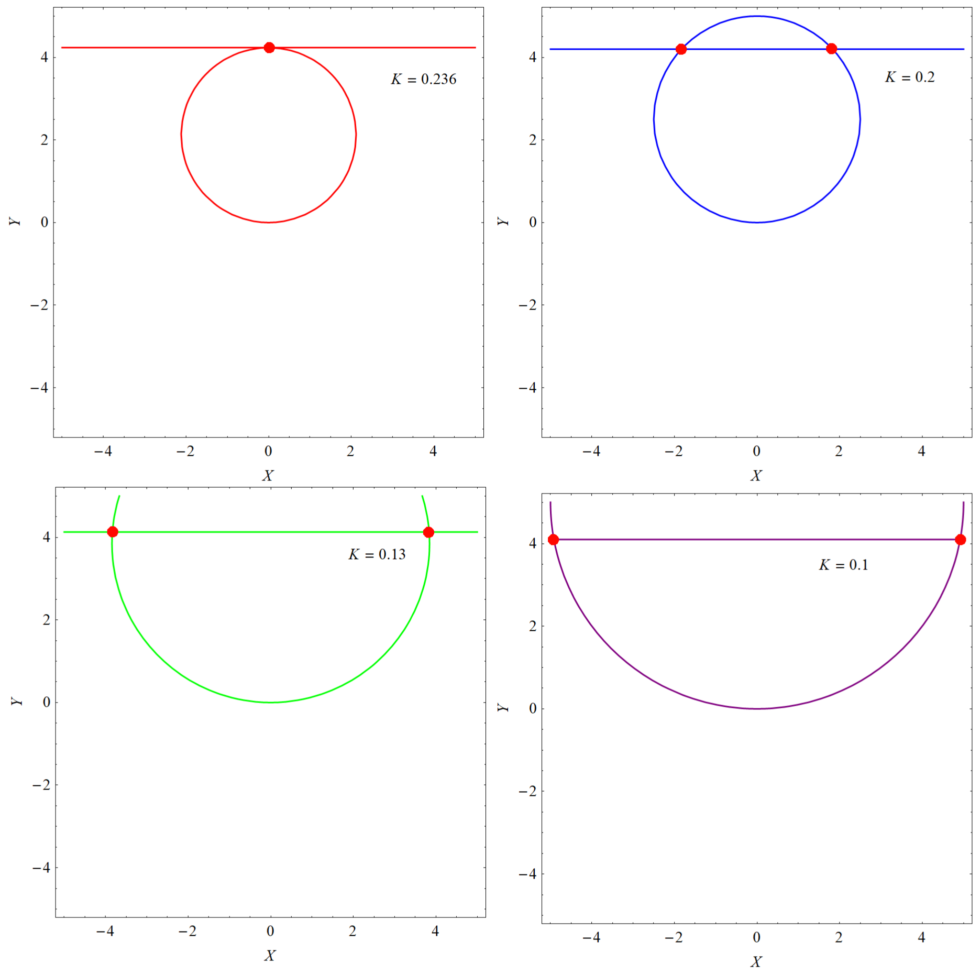
$$\text{Center: } \left(0, \frac{1}{2K}\right) \quad \text{Radius: } \frac{1}{2|K|}.$$

The Red Streamline

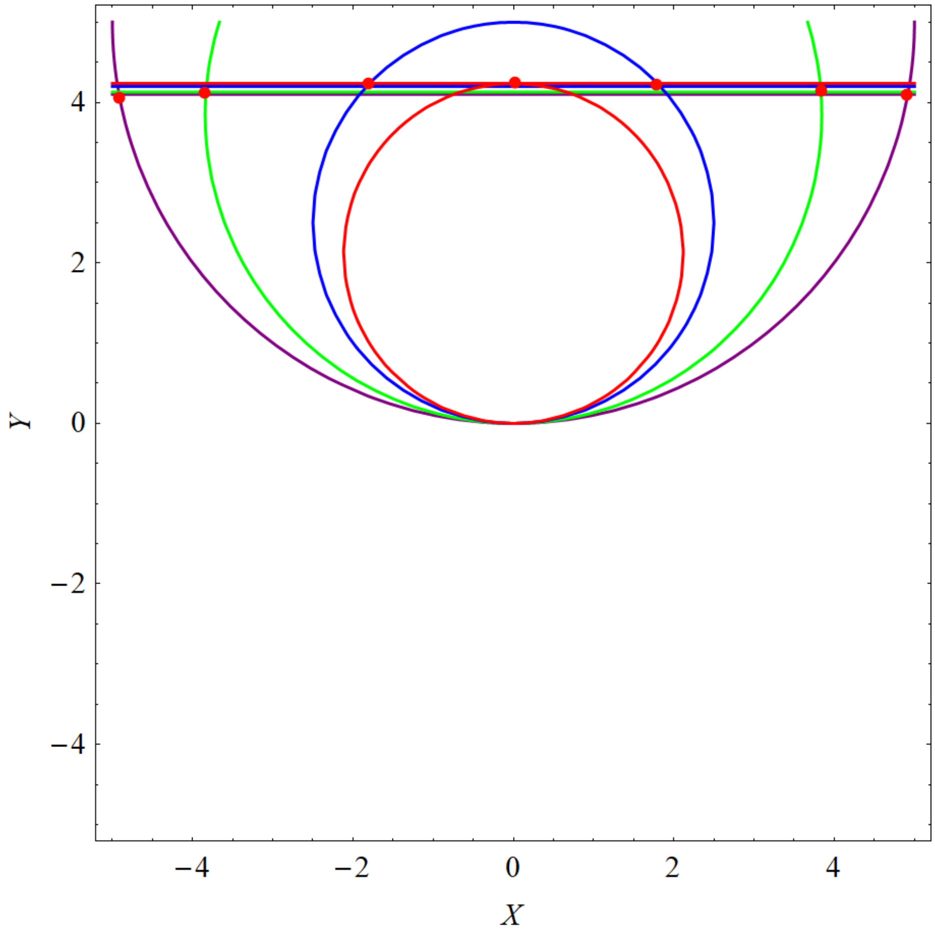
To obtain the red streamline shown in the first figure, set $C = -4$

$$Y = K + 4$$
$$Y = K(X^2 + Y^2).$$

and plot these functions for the values of K that they intersect.



Superimpose the graphs.



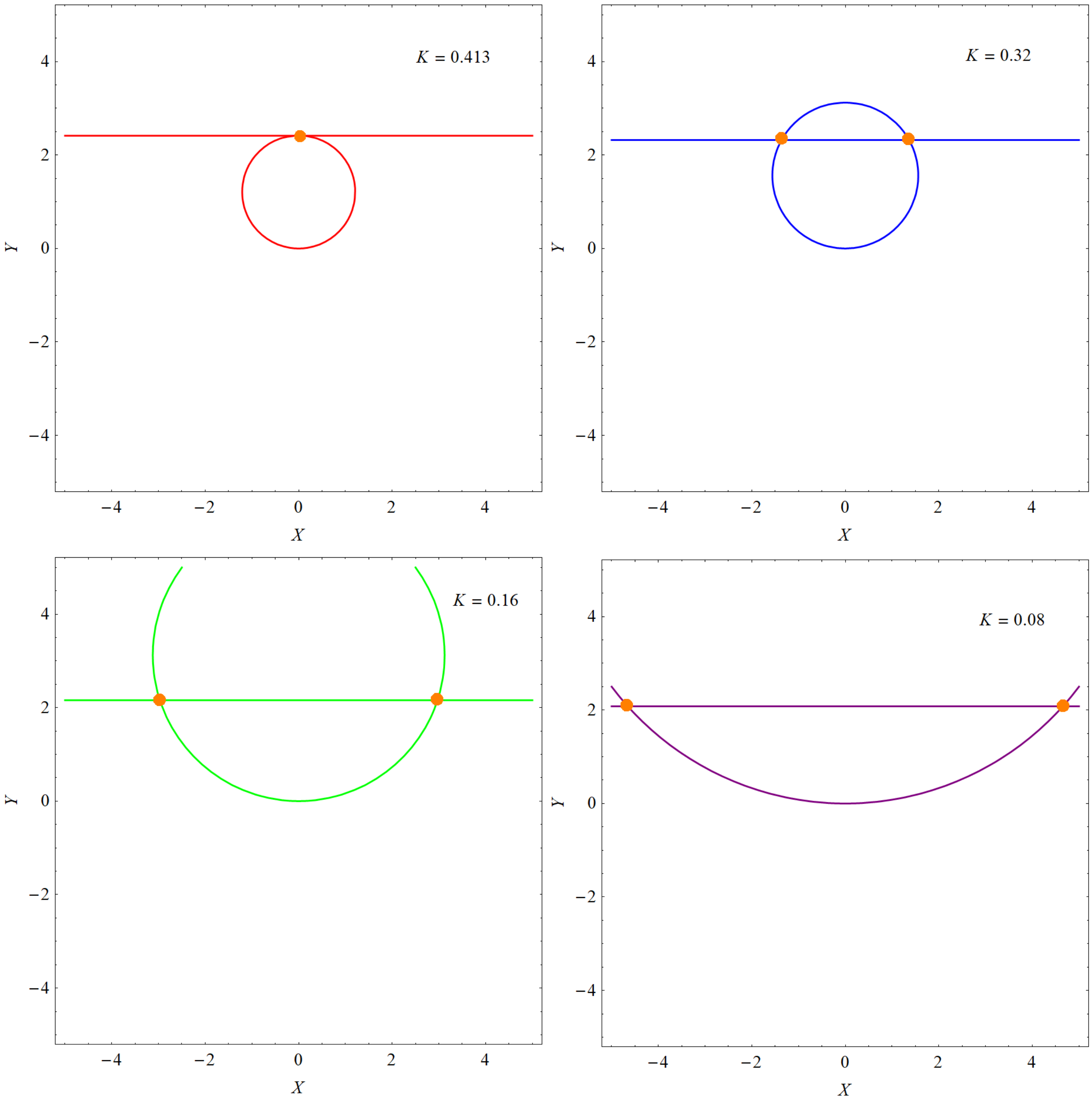
Finally, connect the dots with a smooth curve to obtain the red streamline.

The Orange Streamline

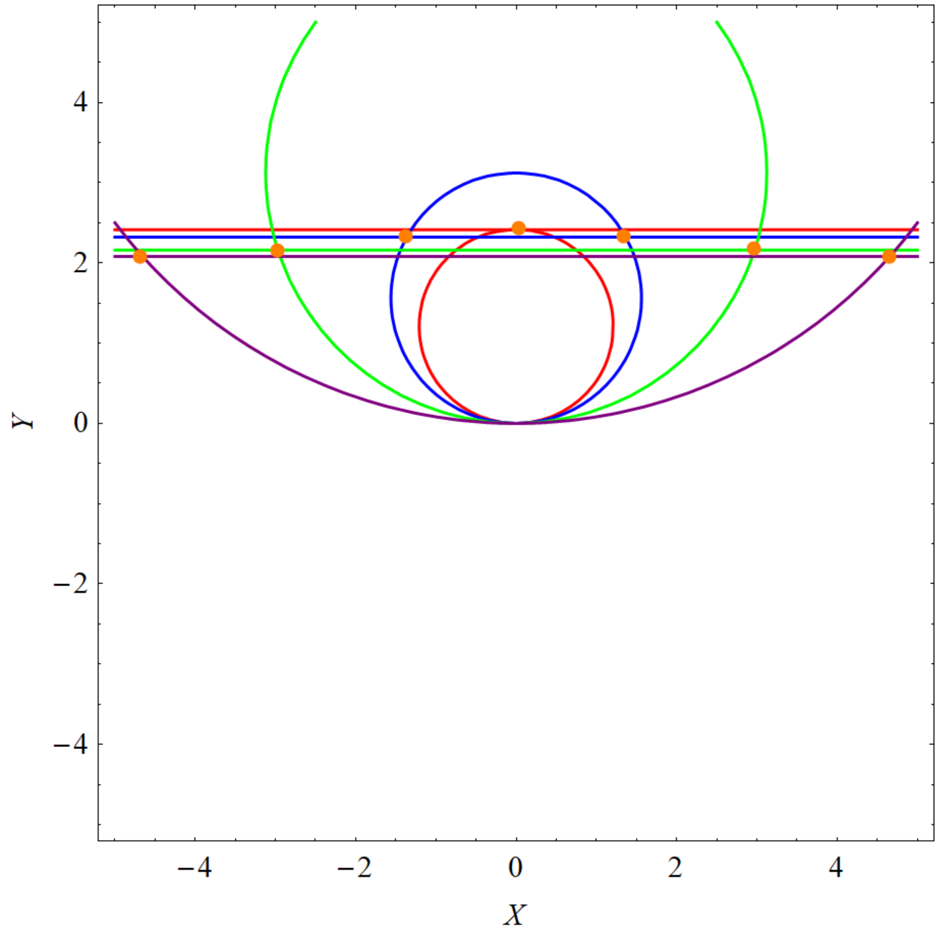
To obtain the orange streamline shown in the first figure, set $C = -2$

$$Y = K + 2$$
$$Y = K(X^2 + Y^2).$$

and plot these functions for the values of K that they intersect.



Superimpose the graphs.



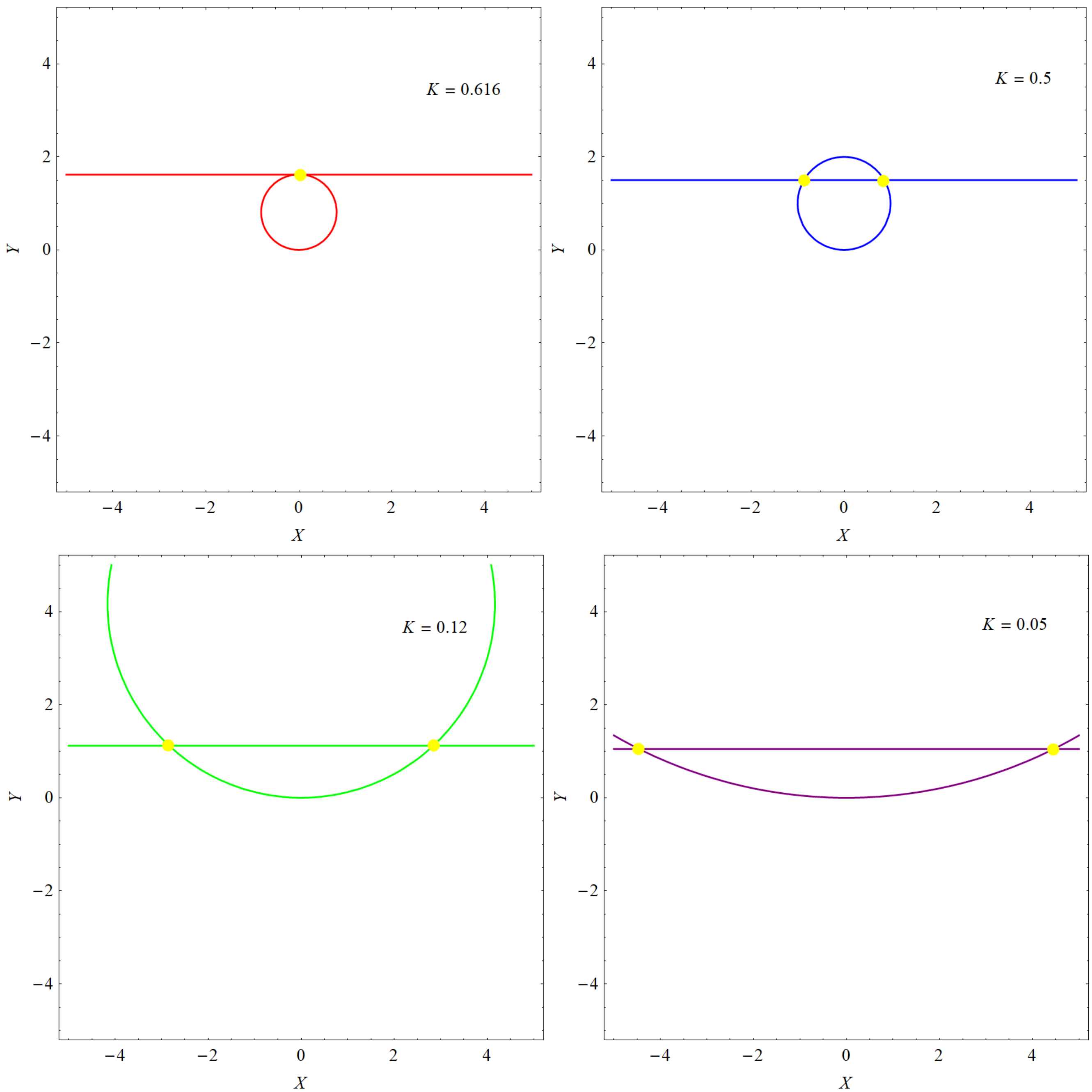
Finally, connect the dots with a smooth curve to obtain the orange streamline.

The Yellow Streamline

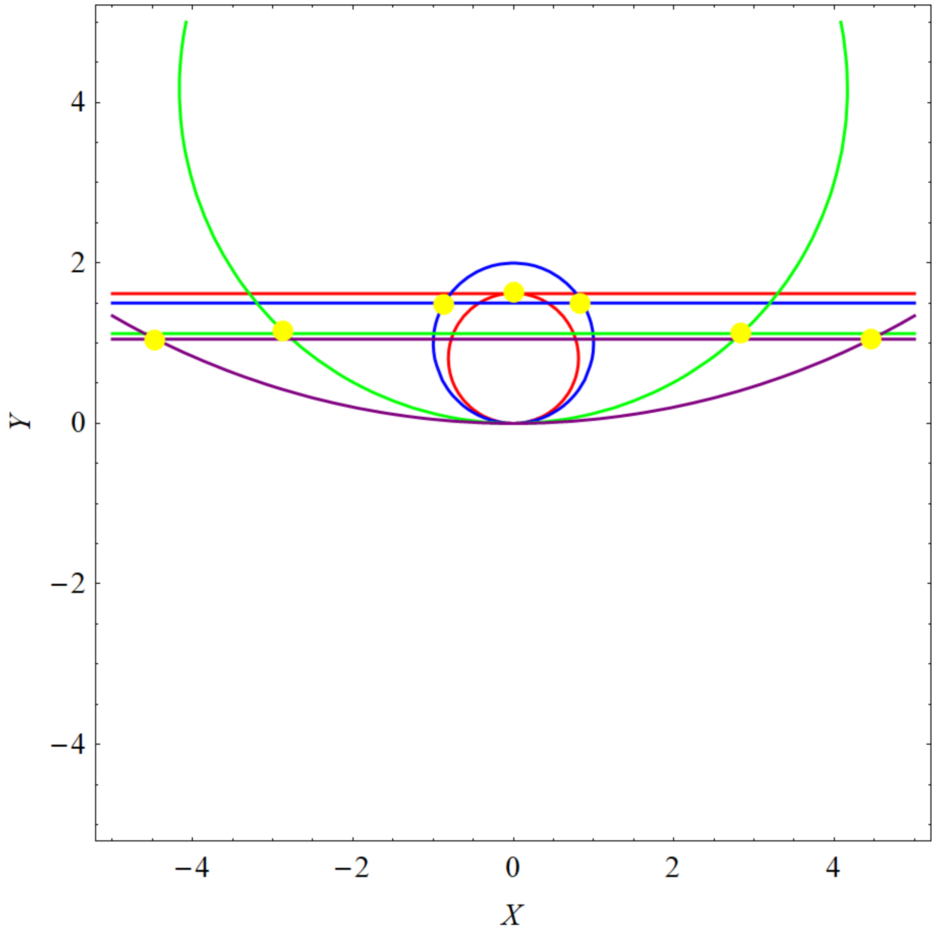
To obtain the yellow streamline shown in the first figure, set $C = -1$

$$Y = K + 1$$
$$Y = K(X^2 + Y^2).$$

and plot these functions for the values of K that they intersect.



Superimpose the graphs.



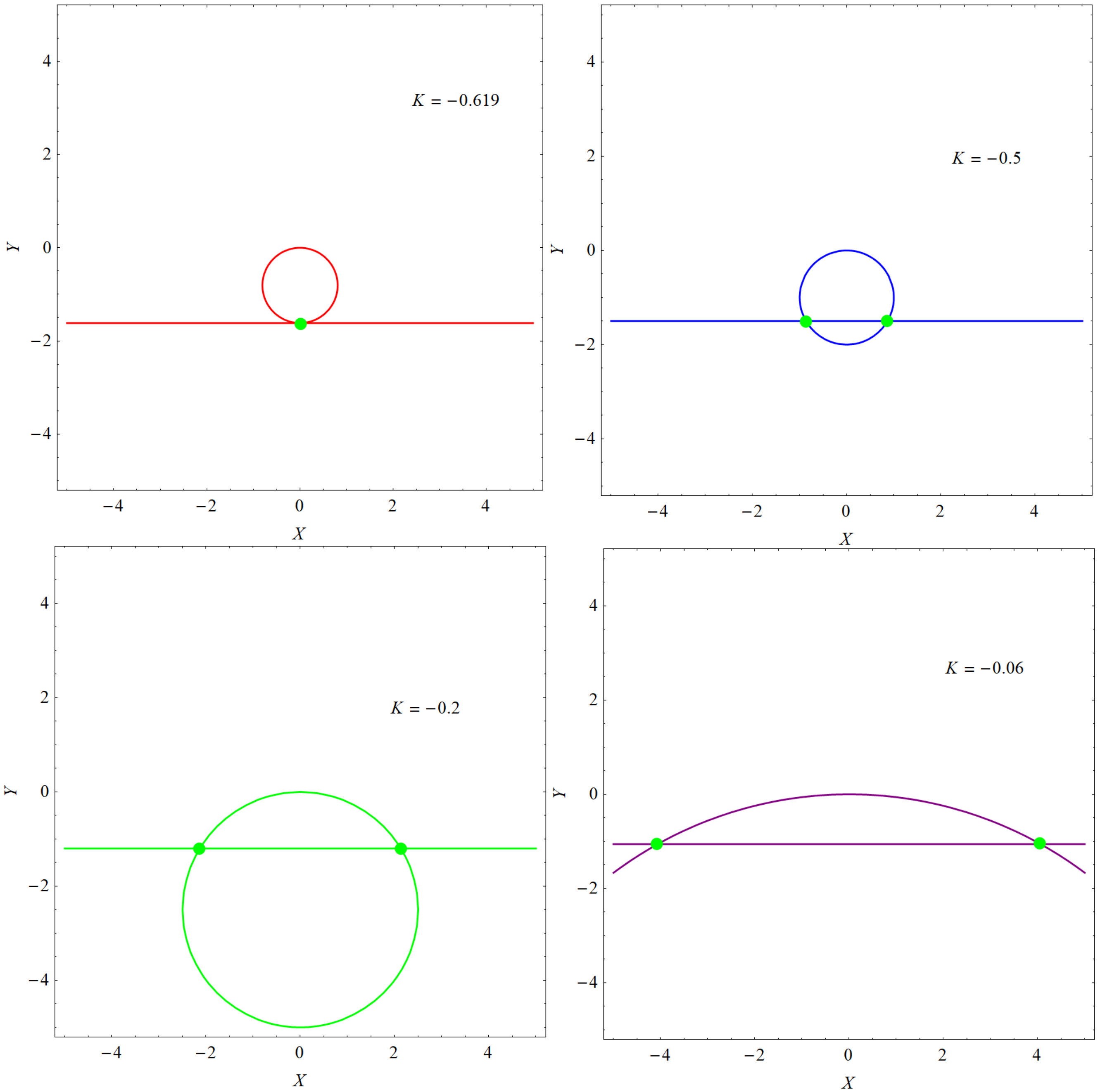
Finally, connect the dots with a smooth curve to obtain the yellow streamline.

The Green Streamline

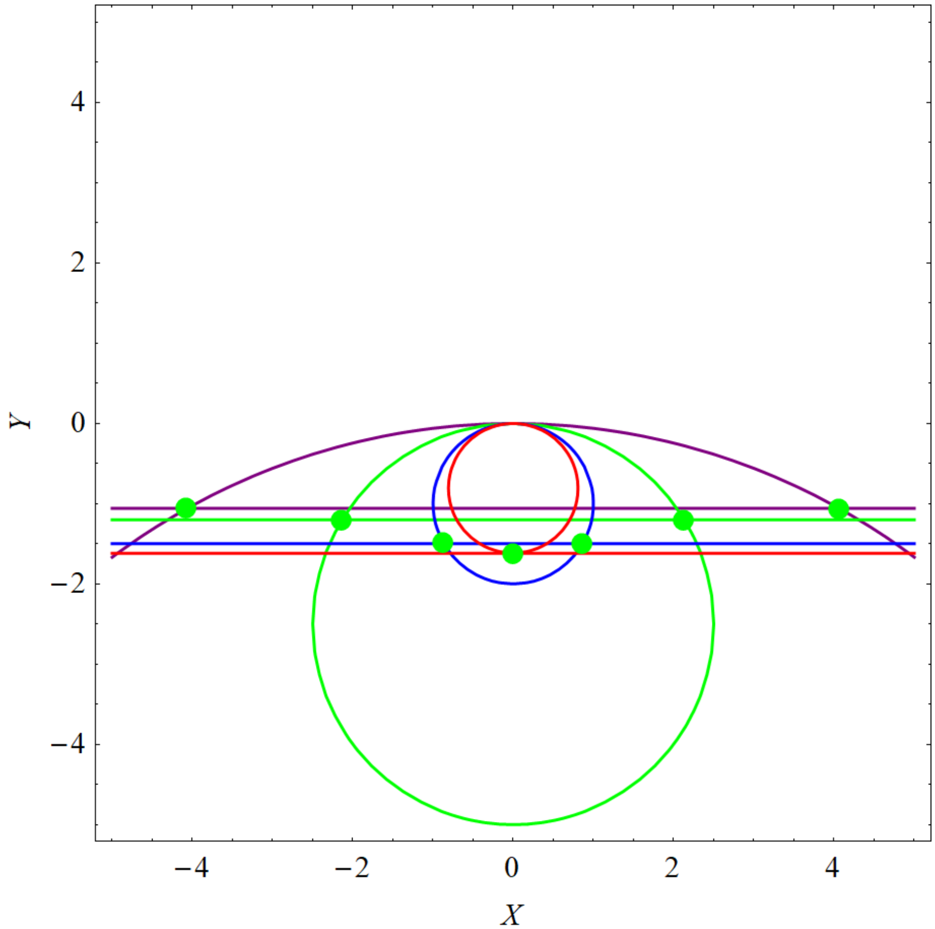
To obtain the green streamline shown in the first figure, set $C = 1$

$$Y = K - 1$$
$$Y = K(X^2 + Y^2).$$

and plot these functions for the values of K that they intersect.



Superimpose the graphs.



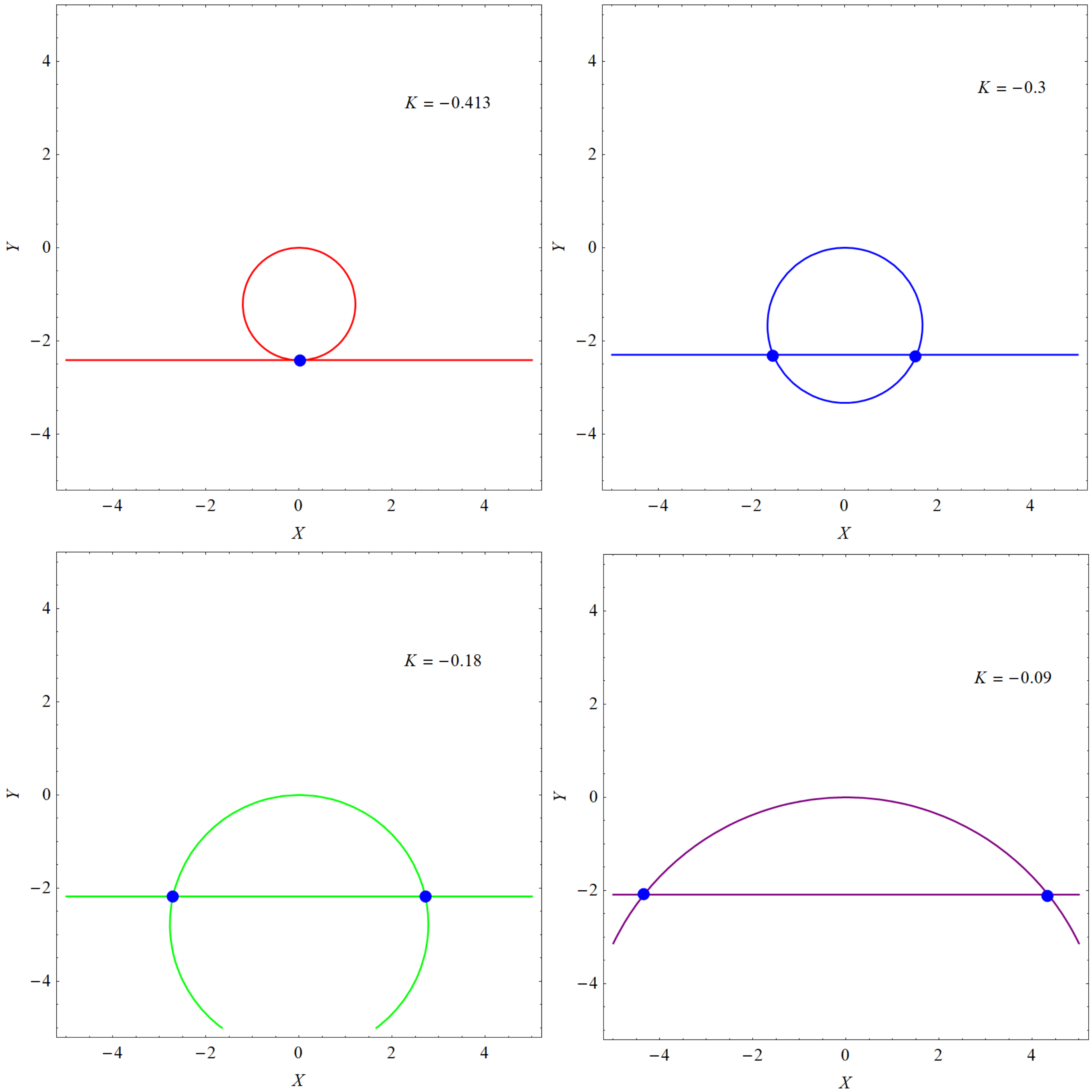
Finally, connect the dots with a smooth curve to obtain the green streamline.

The Blue Streamline

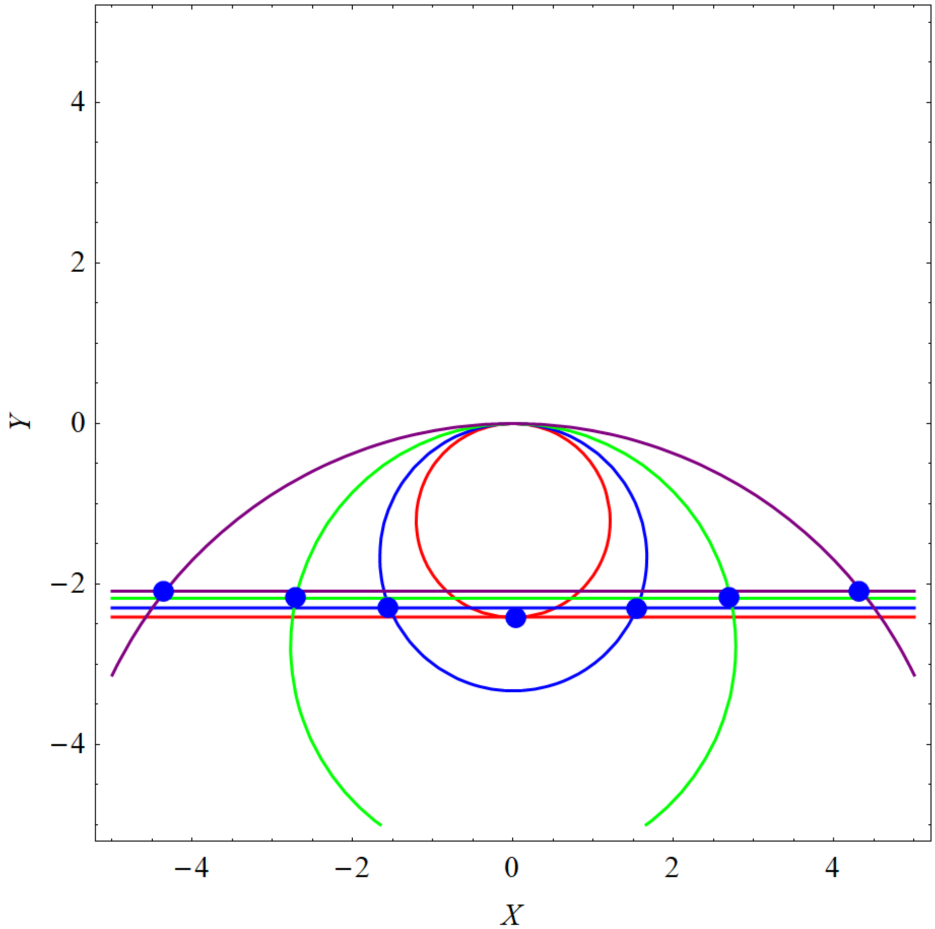
To obtain the blue streamline shown in the first figure, set $C = 2$

$$Y = K - 2$$
$$Y = K(X^2 + Y^2).$$

and plot these functions for the values of K that they intersect.



Superimpose the graphs.



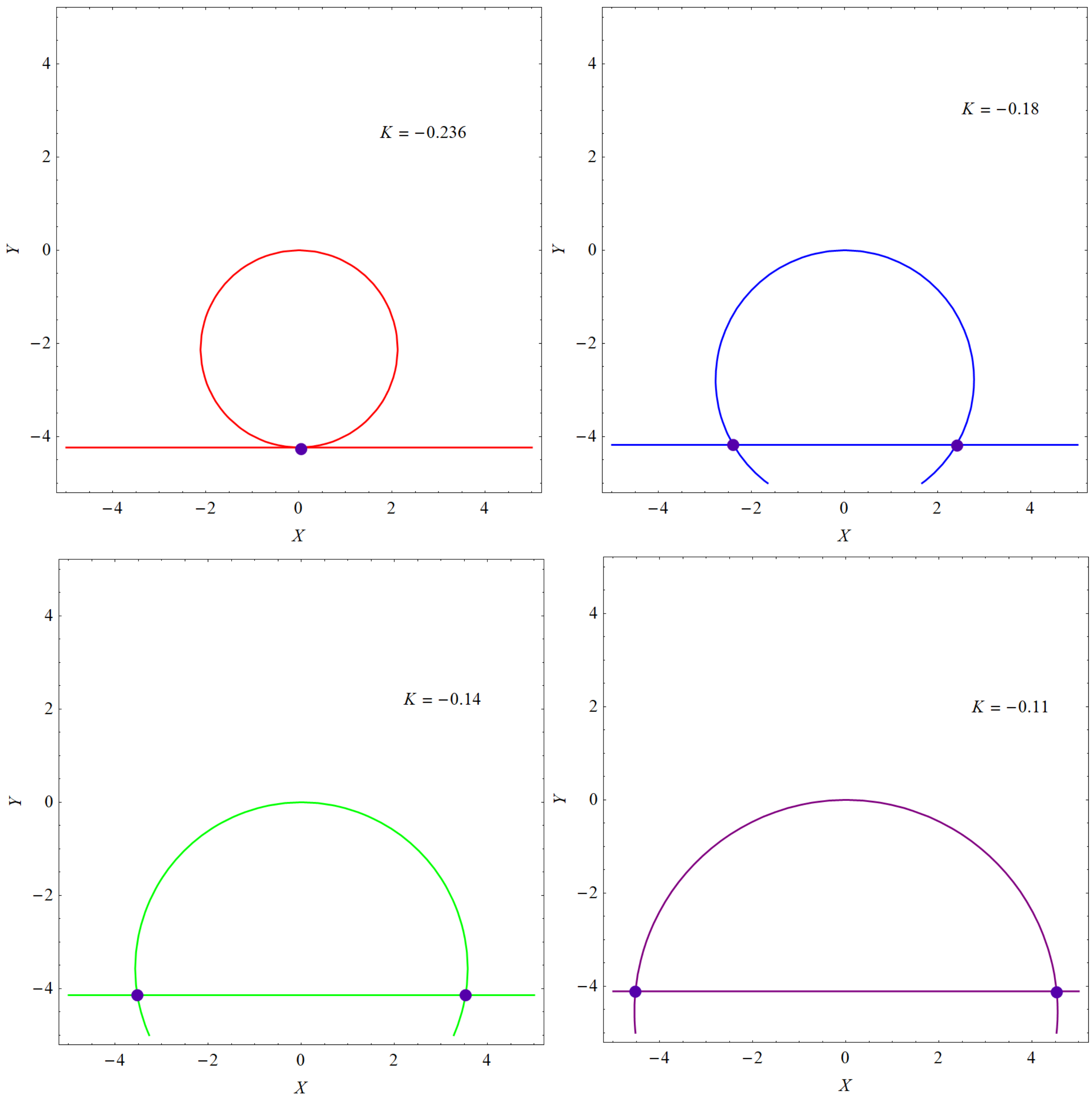
Finally, connect the dots with a smooth curve to obtain the blue streamline.

The Purple Streamline

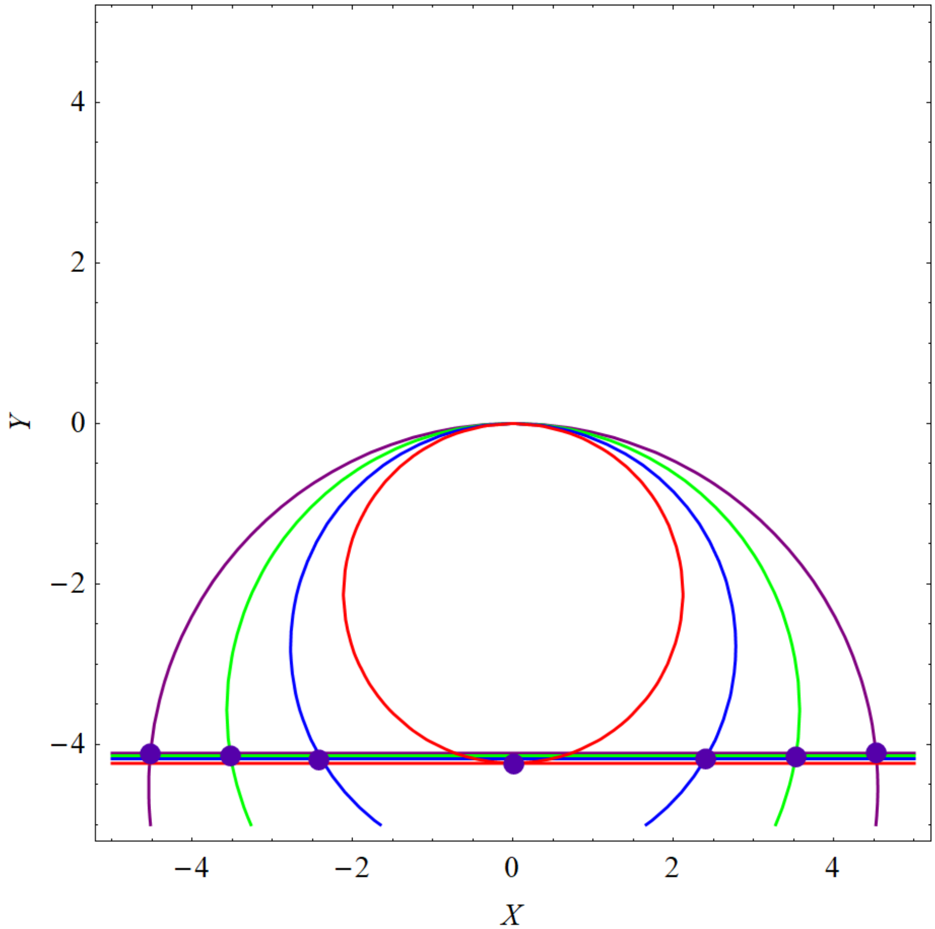
To obtain the purple streamline shown in the first figure, set $C = 4$

$$Y = K - 4$$
$$Y = K(X^2 + Y^2).$$

and plot these functions for the values of K that they intersect.



Superimpose the graphs.



Finally, connect the dots with a smooth curve to obtain the purple streamline.