Problem 4A.6

Use of boundary-layer formulas. Air at 1 atm and 20°C flows tangentially on both sides of a thin, smooth flat plate of width $W = 10$ ft, and of length $L = 3$ ft in the direction of the flow. The velocity outside the boundary layer is constant at 20 ft/s.

(a) Compute the local Reynolds number $\text{Re}_x = xv_\infty/\nu$ at the trailing edge.

(b) Assuming laminar flow, compute the approximate boundary-layer thickness, in inches, at the trailing edge. Use the results of Example 4.4-1.

(c) Assuming laminar flow, compute the total drag on the plate in lb_f. Use the results of Examples 4.4-1 and 2.

Solution

Part (a)

The trailing edge is located at $x = 3$ ft. For tangential laminar flow along a flat plate, the velocity outside the boundary layer is $v_\infty = 20$ ft/s. From Table 1.1-2 on page 14, the kinematic viscosity of air at 20°C is $\nu = 0.1505\ \text{cm}^2/\text{s}$.

$$\text{Re}_x = \frac{xv_\infty}{\nu} = \frac{(3\text{ ft})\left(20\ \frac{\text{ft}}{\text{s}}\right)}{0.1505\ \frac{\cancel{\text{cm}^2}}{\text{s}} \times \left(\frac{1\ \cancel{\text{in}}}{2.54\ \cancel{\text{cm}}}\right)^2 \times \left(\frac{1\text{ ft}}{12\ \cancel{\text{in}}}\right)^2} \approx 4 \times 10^5$$

Because the local Reynolds number is much greater than 1, the formula for the drag force is expected to be very accurate.

Part (b)

The boundary layer thickness is given by Eq. 4.4-17 on page 137.

$$\delta(x) = \sqrt{\frac{280}{13}\frac{\nu x}{v_\infty}} \tag{4.4-17}$$

$$= \sqrt{\frac{280}{13}\frac{0.1505\ \frac{\cancel{\text{cm}^2}}{\text{s}} \times \left(\frac{1\ \cancel{\text{in}}}{2.54\ \cancel{\text{cm}}}\right)^2 \times \left(\frac{1\text{ ft}}{12\ \cancel{\text{in}}}\right)^2 (3\text{ ft})}{20\ \frac{\text{ft}}{\text{s}}}}$$

$$\approx 0.02288\ \cancel{\text{ft}} \times \frac{12\text{ in}}{1\ \cancel{\text{ft}}}$$

$$\approx 0.3\text{ in}$$

Part (c)

Eq. 4.4-19 and Eq. 4.4-30 give the approximate and exact drag force, respectively, on a plate wetted on both sides. Use the exact formula just because.

$$F_x = 1.328\sqrt{\rho\mu LW^2 v_\infty^3} \tag{4.4-30}$$

$$= 1.328\sqrt{\frac{\rho}{\mu}\mu^2 LW^2 v_\infty^3}$$

$$= 1.328\sqrt{\frac{\mu^2 LW^2 v_\infty^3}{\nu}}$$

The viscosity of air at 20°C is also given in Table 1.1-2 on page 14. Do the calculation in SI units and then change to lb_f at the end using the conversion factor in Table F.3-1 on page 868.

$$
\begin{aligned}
F_x &= 1.328\sqrt{\frac{\left(0.01813\ \cancel{\text{mPa}}\cdot\text{s}\times\frac{1\ \text{Pa}}{1000\ \cancel{\text{mPa}}}\right)^2\left(3\ \cancel{\text{ft}}\times\frac{1\ \text{m}}{3.28\ \cancel{\text{ft}}}\right)\left(10\ \cancel{\text{ft}}\times\frac{1\ \text{m}}{3.28\ \cancel{\text{ft}}}\right)^2\left(20\ \frac{\cancel{\text{ft}}}{\text{s}}\times\frac{1\ \text{m}}{3.28\ \cancel{\text{ft}}}\right)^3}{0.1505\ \frac{\cancel{\text{cm}^2}}{\text{s}}\times\left(\frac{1\ \text{m}}{100\ \cancel{\text{cm}}}\right)^2}} \\
&\approx 0.2725\ \cancel{\text{N}}\times\frac{2.24881\times10^{-1}\ \text{lb}_f}{1\ \cancel{\text{N}}} \\
&\approx 0.06\ \text{lb}_f
\end{aligned}
$$