Problem 4D.3

Flows in the disk-and-tube system (Fig. 4D.3).[9]

(a) A fluid in a circular tube is caused to move tangentially by a tightly fitting rotating disk at the liquid surface at $z = 0$; the bottom of the tube is located at $z = L$. Find the steady-state velocity distribution $v_\theta(r, z)$, when the angular velocity of the disk is Ω. Assume that creeping flow prevails throughout, so that there is no secondary flow. Find the limit of the solution as $L \to \infty$.

(b) Repeat the problem for the unsteady flow. The fluid is at rest before $t = 0$, and the disk suddenly begins to rotate with an angular velocity Ω at $t = 0$. Find the velocity distribution $v_\theta(r, z, t)$ for a column of fluid of height L. Then find the solution for the limit as $L \to \infty$.

(c) If the disk is oscillating sinusoidally in the tangential direction with amplitude Ω_0, obtain the velocity distribution in the tube when the "oscillatory steady state" has been attained. Repeat the problem for a tube of infinite length.

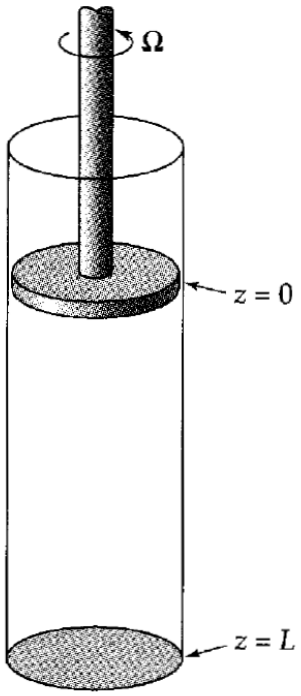

Fig. 4D.3. Rotating disk in a circular tube.

[9] W. Hort, *Z. tech. Phys.*, **10**, 213 (1920); C. T. Hill, J. D. Huppler, and R. B. Bird, *Chem. Engr. Sci.*, **21**, 815–817 (1966).