Problem 5A.1

Pressure drop needed for laminar-turbulent transition. A fluid with viscosity 18.3 cp and density 1.32 g/cm^3 is flowing in a long horizontal tube of radius 1.05 in. (2.67 cm). For what pressure gradient will the flow become turbulent?

Answer: 26 psi/mi (1.1×10^5 Pa/km)

Solution

The fluid flow in a circular tube normally becomes turbulent when the Reynolds number, defined on page 52 to be

$$\text{Re} = \frac{D\langle v_z\rangle\rho}{\mu},$$

exceeds 2100. Use formula (2.3-20) on page 51 to write the average velocity in terms of the modified pressure difference.

$$\begin{aligned}\text{Re} &= \frac{D\rho}{\mu}\frac{(\mathscr{P}_0 - \mathscr{P}_L)R^2}{8\mu L} \\ &= \frac{(2R)\rho}{\mu}\frac{(\mathscr{P}_0 - \mathscr{P}_L)R^2}{8\mu L} \\ &= \frac{(\mathscr{P}_0 - \mathscr{P}_L)\rho R^3}{4\mu^2 L}\end{aligned}$$

Because the tube is horizontal, gravity is irrelevant here, which means the pressure p can be used instead of the modified pressure.

$$\text{Re} = \frac{(p_0 - p_L)\rho R^3}{4\mu^2 L}$$

Solve for the pressure gradient, plug in the numbers, and convert to SI units using the conversion factors on page 870.

$$\begin{aligned}\frac{p_0 - p_L}{L} &= \frac{4\mu^2}{\rho R^3}\text{Re} \\ &= \frac{4\left(18.3\ \cancel{\text{cp}} \times \frac{10^{-3}\ \text{Pa}\cdot\text{s}}{1\ \cancel{\text{cp}}}\right)^2}{\left[1.32\ \frac{\cancel{\text{g}}}{\cancel{\text{cm}^3}} \times \frac{1\ \text{kg}}{1000\ \cancel{\text{g}}} \times \left(\frac{100\ \cancel{\text{cm}}}{1\ \text{m}}\right)^3\right]\left(2.67\ \cancel{\text{cm}} \times \frac{1\ \text{m}}{100\ \cancel{\text{cm}}}\right)^3}(2100) \\ &\approx 1.1 \times 10^2\ \frac{(\text{Pa}\cdot\text{s})^2}{\text{kg}} \\ &= 1.1 \times 10^2\ \frac{\text{Pa}\cdot\cancel{\text{s}}}{\cancel{\text{kg}}} \times \frac{\cancel{\text{kg}}}{\text{m}\cdot\cancel{\text{s}}} \\ &= 1.1 \times 10^2\ \frac{\text{Pa}}{\cancel{\text{m}}} \times \frac{10^3\ \cancel{\text{m}}}{1\ \text{km}} \\ &= 1.1 \times 10^5\ \frac{\text{Pa}}{\text{km}} \\ &= 1.1 \times 10^5\ \frac{\cancel{\text{Pa}}}{\cancel{\text{km}}} \times \frac{1.4504 \times 10^{-4}\ \frac{\text{lb}_f}{\text{in}^2}}{1\ \cancel{\text{Pa}}} \times \frac{1.6\ \cancel{\text{km}}}{1\ \text{mi}} \\ &\approx 26\ \frac{\text{psi}}{\text{mi}}\end{aligned}$$