

**Problem 1.31**

Solve the following differential equations:

(a)  $y' = y/x + 1/y$ ;

(b)  $y' = xy/(x^2 + y^2)$ ;

(c)  $y' = x^2 + 2xy + y^2$ ;

(d)  $yy'' = 2(y')^2$ ;

(e)  $y' = (1 + x)y^2/x^2$ ;

(f)  $x^2y' + xy + y^2 = 0$ ;

(g)  $xy' = y(1 - \ln x + \ln y)$ ;

(h)  $(x + y^2) + 2(y^2 + y + x - 1)y' = 0$ , using an integrating factor of the form  $I(x, y) = e^{ax+by}$ ;

(i)  $-xy' + y = xy^2$  [ $y(1) = 1$ ];

(j)  $y'' - (1 + x)^{-2}(y')^2 = 0$  [ $y(0) = y'(0) = 1$ ];

(k)  $2xyy' + y^2 - x^2 = 0$ ;

(l)  $y'' = (y')^2e^{-y}$  (if  $y' = 1$  at  $y = \infty$ , find  $y'$  at  $y = 0$ );

(m)  $y' = |y - x|$  [if  $y(0) = \frac{1}{2}$ , find  $y(1)$ ];

(n)  $xy' = y + xe^{y/x}$ ;

(o)  $y' = (x^4 - 3x^2y^2 - y^3)/(2x^3y + 3y^2x)$ ;

(p)  $(x^2 + y^2)y' = xy$ ,  $y(e) = e$ ;

(q)  $y'' + 2y'y = 0$  [ $y(0) = y'(0) = -1$ ];

(r)  $x^2y'' + xy' - y = 3x^2$  [ $y(1) = y(2) = 1$ ];

(s)  $y^3(y')^2y'' = -\frac{1}{2}$  [ $y(0) = y'(0) = 1$ ]

(t)  $xy' = y + \sqrt{xy}$ ;

(u)  $(xy)y' + y \ln y = 2xy$  [try an integrating factor of the form  $I = I(y)$ ]; [TYPO: The first term should be  $xy'$ ]

(v)  $(x \sin y + e^y)y' = \cos y$ ;

(w)  $(x + y^2x)y' + x^2y^3 = 0$  [ $y(1) = 1$ ];

(x)  $(x - 1)(x - 2)y' + y = 2$  [ $y(0) = 1$ ];

(y)  $y' = 1/(x + e^y)$ ;

(z)  $xy' + y = y^2x^4$ .