

Problem 1.31

Solve the following differential equations:

$$(e) \quad y' = (1+x)y^2/x^2;$$

Solution

This ODE can be solved by separation of variables.

$$\frac{dy}{dx} = \frac{1+x}{x^2} y^2$$

Split up the fraction on the right side with x .

$$\frac{dy}{y^2} = \left(\frac{1}{x^2} + \frac{1}{x} \right) dx$$

Integrate both sides.

$$-\frac{1}{y} = -\frac{1}{x} + \ln|x| + C$$

Combine the terms on the right side.

$$-\frac{1}{y} = \frac{-1 + x \ln|x| + Cx}{x}$$

Invert both sides and multiply both sides by -1 .

$$y = \frac{x}{1 - x \ln|x| - Cx}$$

Introduce a new arbitrary constant A to eliminate the minus sign. Therefore,

$$y(x) = \frac{x}{1 - x \ln|x| + Ax}.$$