Problem 1.31

Solve the following differential equations:

(*q*) $y'' + 2y'y = 0$ $[y(0) = y'(0) = -1]$;

Solution

The left side of the ODE can be written as follows.

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) + \frac{d}{dx}(y^2) = 0$$

Integrate both sides with respect to x.

$$\frac{dy}{dx} + y^2 = A$$

We can determine A by using the provided initial conditions. When $x = 0$, y and dy/dx are equal to -1.

$$-1 + (-1)^2 = A \quad \rightarrow \quad A = 0,$$

so the ODE simplifies to

$$\frac{dy}{dx} + y^2 = 0.$$

Move y^2 over to the right side.

$$\frac{dy}{dx} = -y^2$$

This ODE can be solved by separation of variables.

$$y^{-2}\,dy = -dx$$

Integrate both sides.

$$-\frac{1}{y} = -x + B$$

Plug in the initial conditions once again to determine B.

$$1 = B$$

So we have

$$-\frac{1}{y} = -x + 1$$

Multiply both sides by -1.

$$\frac{1}{y} = x - 1$$

Invert both sides to solve for y. Therefore,

$$y(x) = \frac{1}{x-1}.$$

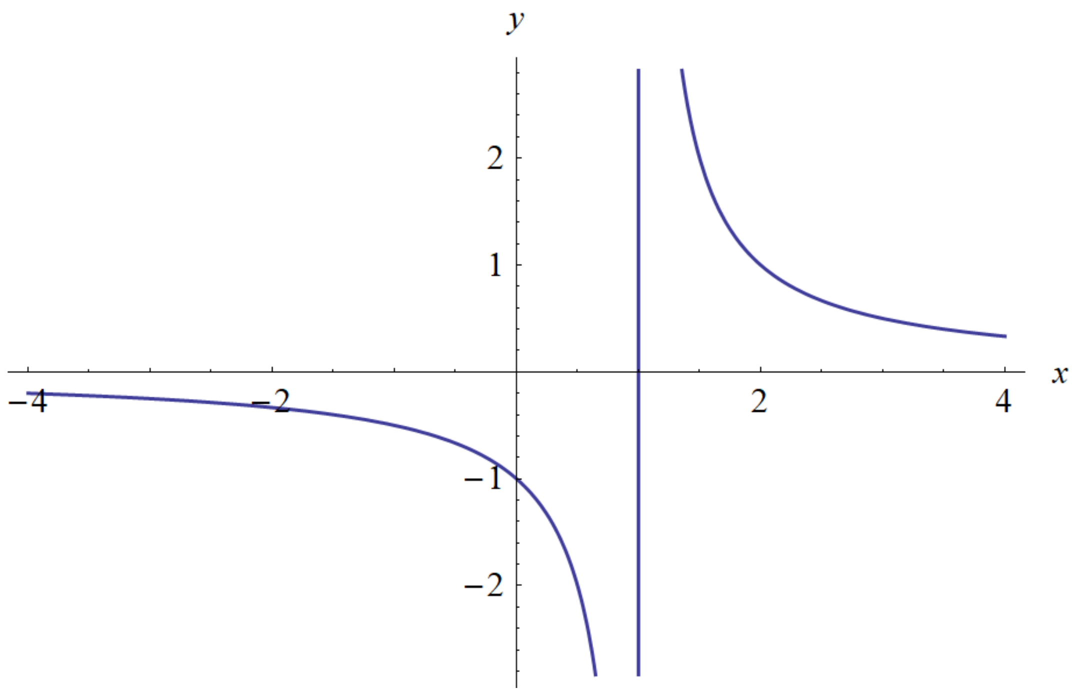

Figure 1: Plot of the solution for $-4 < x < 4$.