Problem 3.11

$$\begin{aligned} \text{If} \quad u &= 1 + \frac{x^3}{3!} + \frac{x^6}{6!} + \cdots, \\ v &= \frac{x}{1!} + \frac{x^4}{4!} + \frac{x^7}{7!} + \cdots, \\ w &= \frac{x^2}{2!} + \frac{x^5}{5!} + \frac{x^8}{8!} + \cdots, \end{aligned}$$

prove that

$$u^3 + v^3 + w^3 - 3uvw = 1$$

(Putnam Exam 1939).