Problem 1

In each of Problems 1 through 6, draw a direction field for the given differential equation. Based on the direction field, determine the behavior of y as $t \to \infty$. If this behavior depends on the initial value of y at $t = 0$, describe the dependency.

$$y' = 3 - 2y$$

Solution

The direction field is a two-dimensional vector field that shows what the direction of the solution is at every point in a region. Every solution to the differential equation is a curve drawn such that the direction field vectors are tangent to it at every point.

$$\langle dt, dy \rangle = \left\langle 1, \frac{dy}{dt} \right\rangle dt = \langle 1, 3 - 2y \rangle \, dt$$

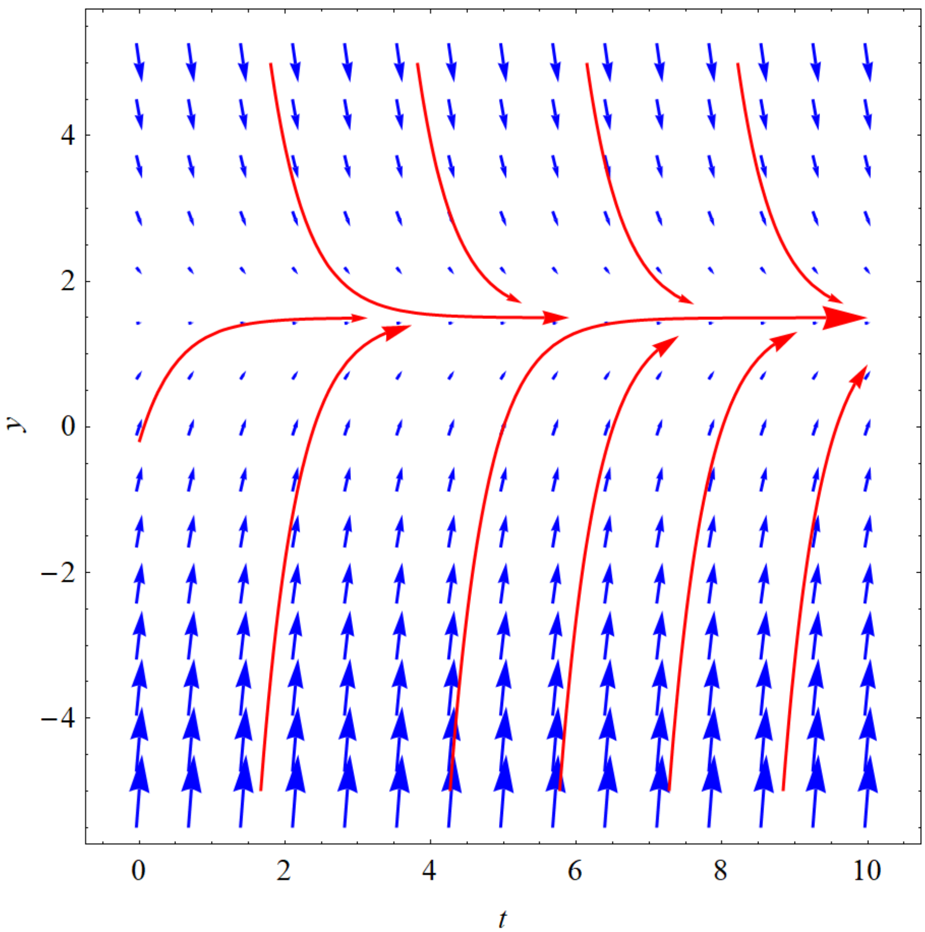

Figure 1: In blue are the direction field vectors and in red are possible solutions to the differential equation, depending what the initial condition is. The nonequilibrium solutions appear to converge to $y = 1.5$ as $t \to \infty$.

The (stable) equilibrium solution is found by setting $y' = 0$ in the differential equation and solving the resulting equation for y.

$$0 = 3 - 2y$$

$$y = \frac{3}{2}$$