

### Problem 12

In each of Problems 11 through 14, draw a direction field for the given differential equation. Based on the direction field, determine the behavior of  $y$  as  $t \rightarrow \infty$ . If this behavior depends on the initial value of  $y$  at  $t = 0$ , describe this dependency. Note that in these problems the equations are not of the form  $y' = ay + b$ , and the behavior of their solutions is somewhat more complicated than for the equations in the text.

$$y' = -y(5 - y)$$

#### Solution

The direction field is a two-dimensional vector field that shows what the direction of the solution is at every point in a region. Every solution to the differential equation is a curve drawn such that the direction field vectors are tangent to it at every point.

$$\langle dt, dy \rangle = \left\langle 1, \frac{dy}{dt} \right\rangle dt = \langle 1, -y(5 - y) \rangle dt$$

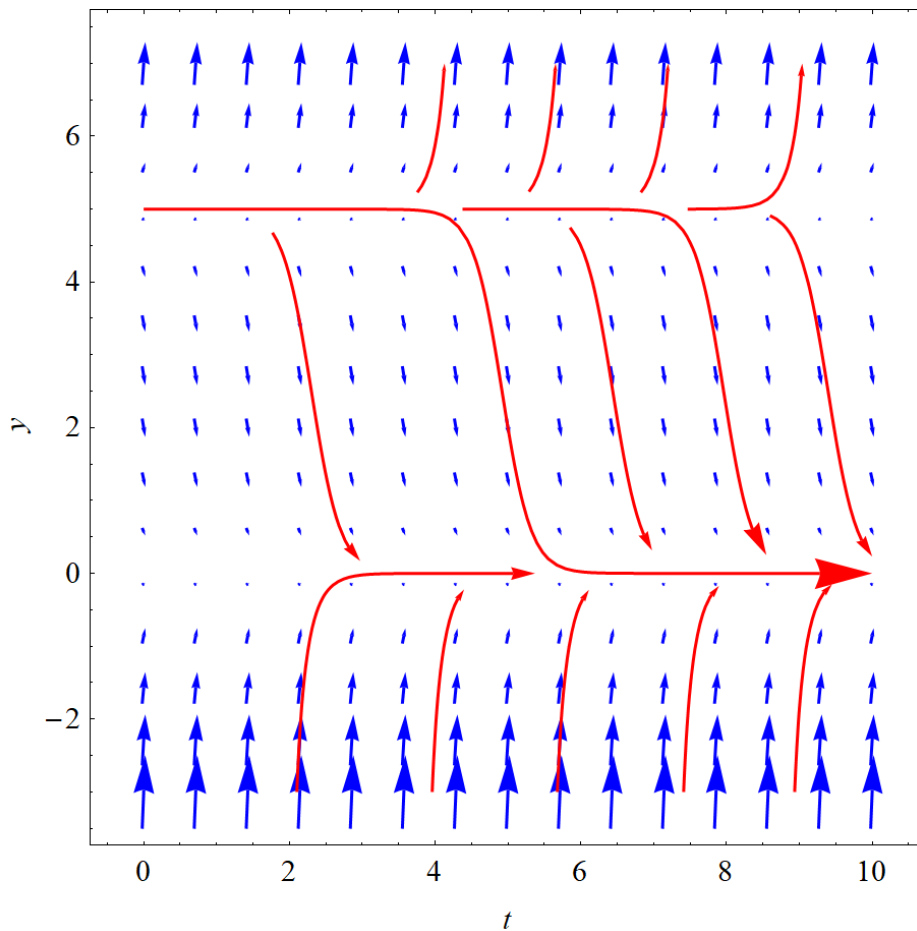


Figure 1: In blue are the direction field vectors and in red are possible solutions to the differential equation, depending what the initial condition is.

The equilibrium solutions are found by setting  $y' = 0$  in the differential equation and solving the resulting equation for  $y$ .

$$0 = -y(5 - y)$$
$$y = 0 \quad \text{or} \quad y = 5$$

Notice from Figure 1 that the behavior of  $y$  as  $t \rightarrow \infty$  depends on what the initial condition is. If  $y(t = 0) > 5$ , then the solution diverges to  $\infty$  as  $t \rightarrow \infty$ . If  $0 < y(t = 0) < 5$ , then the solution converges to  $y = 0$  as  $t \rightarrow \infty$ . If  $y(t = 0) < 0$ , then the solution converges to  $y = 0$  as  $t \rightarrow \infty$ .