Problem 33

In each of Problems 26 through 33, draw a direction field for the given differential equation. Based on the direction field, determine the behavior of y as $t \to \infty$. If this behavior depends on the initial value of y at $t = 0$, describe this dependency. Note that the right sides of these equations depend on t as well as y; therefore, their solutions can exhibit more complicated behavior than those in the text.

$$y' = \frac{1}{6}y^3 - y - \frac{1}{3}t^2$$

Solution

The direction field is a two-dimensional vector field that shows what the direction of the solution is at every point in a region. Every solution to the differential equation is a curve drawn such that the direction field vectors are tangent to it at every point.

$$\langle dt, dy \rangle = \left\langle 1, \frac{dy}{dt} \right\rangle dt = \left\langle 1, \frac{1}{6}y^3 - y - \frac{1}{3}t^2 \right\rangle dt$$

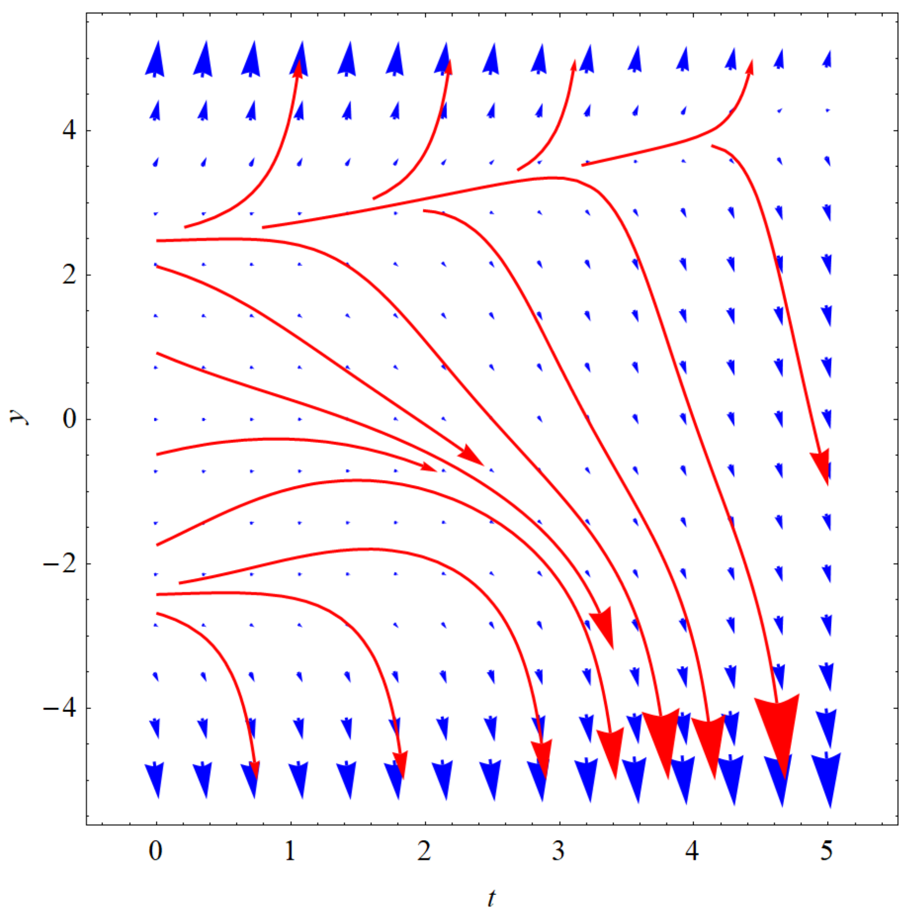

Figure 1: In blue are the direction field vectors and in red are possible solutions to the differential equation, depending what the initial condition is. Above a certain curve in the ty-plane, the solutions diverge to ∞ as $t \to \infty$. Below it, they diverge to $-\infty$ as $t \to \infty$.