Problem 8

In each of Problems 7 through 10, write down a differential equation of the form $dy/dt = ay + b$ whose solutions have the required behavior as $t \to \infty$.

All solutions approach $y = 2/3$.

Solution

dy/dt represents the rate of change of y with respect to t. Setting $dy/dt = 0$ in the differential equation allows us to find the equilibrium solution.

$$0 = ay + b$$

$$y = -\frac{b}{a}$$

a and b need to be chosen so that this ratio evaluates to $2/3$. In addition, a has to be negative so that the nonequilibrium solutions converge as $t \to \infty$. One possible choice is $a = -3$ and $b = 2$.

$$\frac{dy}{dt} = -3y + 2$$

Below in Figure 1 is the direction field for this differential equation along with possible solutions.

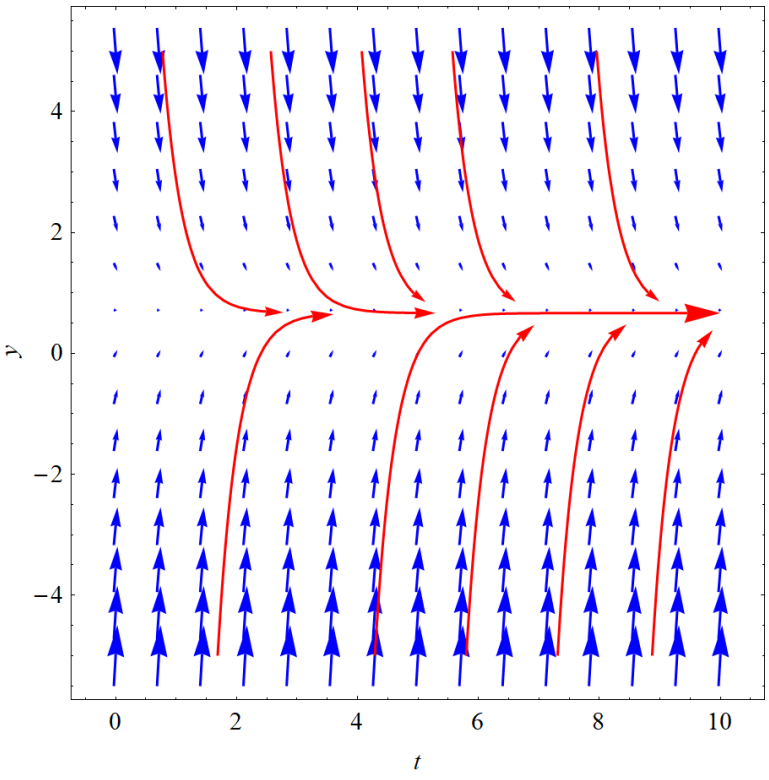

Figure 1: In blue are the direction field vectors and in red are possible solutions to the differential equation, depending what the initial condition is. The nonequilibrium solutions appear to converge to $y = 2/3$ as $t \to \infty$.