

Problem 4

Consider the differential equation $dy/dt = ay - b$.

- (a) Find the equilibrium solution y_e .
 - (b) Let $Y(t) = y - y_e$; thus $Y(t)$ is the deviation from the equilibrium solution. Find the differential equation satisfied by $Y(t)$.
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Solution

Part (a)

At equilibrium the solution does not vary with time, so y_e satisfies

$$0 = ay_e - b.$$

Therefore,

$$y_e = \frac{b}{a}.$$

Part (b)

Solve the substitution for y

$$y = Y + y_e$$

and differentiate both sides with respect to t .

$$\frac{dy}{dt} = \frac{dY}{dt}$$

As a result, the differential equation becomes

$$\begin{aligned} \frac{dy}{dt} = ay - b &\quad \rightarrow \quad \frac{dY}{dt} = a(Y + y_e) - b \\ &= a\left(Y + \frac{b}{a}\right) - b \\ &= aY + b - b. \end{aligned}$$

Therefore,

$$\frac{dY}{dt} = aY.$$